

نام و نام خانوادگی آ. ج. ط. ا. ب. پاسخنامه تشریحی تکلیف شماره ۲۰ کلاس روز بهمن ۱۳۹۷

$$\lim_{x \rightarrow r} \frac{2x - a}{x^2 + ax + b} = \lim_{x \rightarrow r} \frac{2x - a}{(x - r)^2} = \lim_{x \rightarrow r} \frac{2x - a}{x^2 - 2rx + r^2} \rightarrow \begin{matrix} a = -r \\ b = r \end{matrix}$$

$$a + b = -r + r = 0 \quad \checkmark$$

$$\lim_{x \rightarrow \left(\frac{\sqrt{r}}{a}\right)} \frac{x}{x^2 + ax + b} = +\infty \rightarrow \lim_{x \rightarrow \frac{\sqrt{r}}{a}} \frac{x}{x^2 + ax + b} = \lim_{x \rightarrow a(x - \sqrt{r})^2} \frac{x}{x^2 + ax + b}$$

$$= \lim_{x \rightarrow a} \frac{x}{x^2 - 2\sqrt{r}x + r} \rightarrow \begin{matrix} a = -r\sqrt{r} \\ b = r\sqrt{r} \end{matrix} \rightarrow \left[\frac{b}{a}\right] = \left[\frac{r\sqrt{r}}{-r\sqrt{r}}\right] = -1$$

$$\sqrt{r} \approx 1/r \rightarrow \frac{1}{\sqrt{r}} = \frac{10}{1r} \quad \lim_{x \rightarrow \left(\frac{1}{\sqrt{r}}\right)^+} \frac{[-x] + a}{\left|\frac{1}{\sqrt{r}}x + a\right| + a} = +\infty$$

$$\frac{\left[-\frac{10}{1r}\right] + a}{\left|\frac{1}{\sqrt{r}}x + a\right| + a} = \frac{a - 1}{\left|\frac{1}{\sqrt{r}}x + a\right| + a} = +\infty \rightarrow \begin{matrix} +\infty \rightarrow \frac{a - 1}{\frac{1}{\sqrt{r}} + a} = +\infty \rightarrow \frac{1}{\sqrt{r}} + a = 0 \\ -\infty \rightarrow \frac{a - 1}{-\frac{1}{\sqrt{r}} - a + a} = \frac{a - 1}{-\frac{1}{\sqrt{r}}} \end{matrix}$$

$$[a] = \left[-\frac{1}{\sqrt{r}}\right] = -1 \quad \checkmark$$

$$x > \frac{1}{r} \rightarrow x^r < \frac{1}{r} \rightarrow \frac{1}{x^r} > r \rightarrow \frac{-r}{x^r} < -1$$

$$x < \frac{1}{r} \rightarrow x^r > \frac{1}{r} \rightarrow \frac{1}{x^r} < r \rightarrow \frac{r}{x^r} > 1$$

$$\lim_{x \rightarrow \left(\frac{1}{r}\right)^+} \frac{14x - \left[-\frac{r}{x^r}\right]}{25x + \left[\frac{r}{x^r}\right]} = \lim_{x \rightarrow \left(\frac{1}{r}\right)^+} \frac{14x \left(\frac{1}{r}\right) - (-9)}{25x \left(\frac{1}{r}\right) + 11} = \frac{-1 + 9}{-15 + 11} = \frac{8}{-4} = -2$$

$$\lim_{x \rightarrow r^+} \frac{x^r - r}{x^r - [2x^r]} = \lim_{x \rightarrow r} \frac{x^r - r}{x^r - [1^+]} = \lim_{x \rightarrow r} \frac{(x - r)(x + r)}{(x - r)(x^r + r + 2x)}$$

$$= \lim_{x \rightarrow r} \frac{x + r}{x^r + r + 2x} = \frac{r}{r + r + 2r} = \frac{1}{3} \quad \checkmark$$

$$\lim_{x \rightarrow -1} \frac{x^5 + 10x + 15}{1 + 4\sqrt{x}} = \frac{0}{0} \xrightarrow{\text{Hop}} \lim_{x \rightarrow -1} \frac{5x^4 + 10}{\frac{4}{3\sqrt{x}}}$$

$$= \lim_{x \rightarrow -1} \frac{(5x^4 + 10)(-\sqrt{x})}{4} = \frac{-5 \times 1}{4} = -\frac{5}{4}$$

9

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{1 - \cos x} \xrightarrow{\text{L'Hopital}} \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}}}{\sin x} \rightarrow \frac{0}{0}$$

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10

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{x})}{kx^2} = \frac{0}{0} \xrightarrow{\text{L'Hopital}} \lim_{x \rightarrow 0} \frac{-\sin(\sqrt{x})}{2kx} = \frac{0}{0}$$

$$\xrightarrow{\text{Hop}} \lim_{x \rightarrow 0} \frac{-\frac{1}{2}\sin(\sqrt{x})}{2k} = \frac{0}{2k} = 0 \rightarrow \frac{0}{k} = 0$$

11

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{2\sqrt{x-a} + 3(\sqrt{x} - \sqrt{a})}{\sqrt{x-a} \times \sqrt{x+a}} = \lim_{x \rightarrow (\sqrt{a})^+} \frac{2\sqrt{x-a}}{\sqrt{x-a} \times \sqrt{x+a}} + \lim_{x \rightarrow (\sqrt{a})^+} \frac{3(\sqrt{x} - \sqrt{a})}{\sqrt{x-a} \times \sqrt{x+a}}$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{2}{\sqrt{x+a}} + \lim_{x \rightarrow (\sqrt{a})^+} \frac{3(\sqrt{x} - \sqrt{a})}{(\sqrt{x-a})(\sqrt{x+a})(\sqrt{x} + \sqrt{a})} = \frac{2}{\sqrt{4a}} + 0 = \frac{2}{\sqrt{4a}}$$

12

$$\lim_{x \rightarrow -1} \frac{1 - k[x]}{(x-1)(x+1)} = -\infty$$

$\xrightarrow{-1^+} \frac{1+k}{0^-} = -\infty \rightarrow 1+k > 0 \rightarrow k > -1$
 $\xrightarrow{-1^-} \frac{1+k}{0^+} = -\infty \rightarrow 1+k < 0 \rightarrow k < -1$

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(دری می) $(k > -1, (k < -1)) \rightarrow$ جواب دارند

13

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{r+x} - \sqrt{r-x}}{-x \cdot \frac{\sqrt{r}}{r}} \times \frac{(\sqrt{r+x} + \sqrt{r-x})}{(\sqrt{r+x} + \sqrt{r-x})}$$

$$= \lim_{x \rightarrow 0^-} \frac{r+x - r+x}{(-x \frac{\sqrt{r}}{r})(\sqrt{r+x} + \sqrt{r-x})}$$

$$= \lim_{x \rightarrow 0^-} \frac{-x}{(-x \frac{\sqrt{r}}{r})(\sqrt{r+x} + \sqrt{r-x})}$$

$$\frac{-1}{\frac{\sqrt{r}}{r} \times (\sqrt{r} + \sqrt{r})} = \frac{-1}{\sqrt{r} \times 2} = -\frac{1}{2\sqrt{r}}$$