رادمهر معتضدی

آفرین ۲۰

شمارهٔ ۲۰

۱) $\lim_{x\to 2} \frac{2x-5}{x^2+ax+b} = -\infty$ ، $\lim_{x\to 2} 2x-5 = -1<0$ $\Big\}$ $\lim_{x\to 2} x^2+ax+b=0$ باید در همسایگی ۲ مثبت باشد $\Rightarrow x^2+ax+b=(x-2)^2=x^2-4x+4$ ۲

$\Rightarrow a+b=0$

۲) $\lim_{x\to \sqrt[3]{4}} \frac{x}{x^2+ax+b}$ ، $\lim_{x\to\sqrt[3]{4}} (x) > 0$ $\Big\}$ $x^2+ax+b=f(x)$ ، $\lim_{x\to\sqrt[3]{4}} f(x)=0$ در همسایگی مثبت است $\Rightarrow f(x)=(x-\sqrt[3]{4})^2=x^2-2\sqrt[3]{4}x+2\sqrt[3]{2}$ ۲

$\left[\frac{b}{a}\right]=\left[\frac{+2\sqrt[3]{2}}{-2\sqrt[3]{4}}\right]=\left[-\sqrt[3]{\frac{1}{2}}\right]=-1$

۳) $\lim_{x\to(\frac{\sqrt{3}}{3})^+} \frac{[-x]+a}{|\frac{\sqrt{3}}{3}x+a|+a}=\lim \frac{a-1}{|\frac{\sqrt{3}}{3}x+a|+a}=-\infty$ ۲

$\Rightarrow |\frac{\sqrt{3}}{3}\times\frac{\sqrt{3}}{3}+a|+a=0 \to -a=|a+\frac{1}{3}| \Rightarrow a^2=a^2+\frac{2}{3}a+\frac{1}{9} \Rightarrow a=-\frac{1}{6}$

↳ $a<0$

$[a]=-1$

۴) $x>-\frac{1}{2} \to x^2<\frac{1}{4} \to \frac{1}{x^2}>4 \to \frac{-2}{x^2}<-8 \to \left[\frac{-2}{x^2}\right]_{x\to-\frac{1}{2}}=-9$ ۲

↳ $\frac{3}{x^2}>12 \to \left[\frac{3}{x^2}\right]=12$

$\Rightarrow \lim_{x\to -\frac{1}{2}^+} \frac{16x-(-9)}{24x+12}=\lim_{x\to-\frac{1}{2}^+} \frac{16x+9}{24(x+\frac{1}{2})}=+\infty$

۵) $x>2 \to x^3>8 \to [x^3]=8$ ۲

$\lim_{x\to 2} \frac{x^2-4}{x^3-(8)}=\lim_{x\to 2} \frac{x+2}{x^2+2x+4}=\frac{4}{12}=\frac{1}{3}$

6) $\lim\limits_{x\to-1}\dfrac{\overbrace{(x+2)}^{-6}(x+1)}{6(\sqrt[3]{x}+2)}=-\lim\limits_{x\to-1}\dfrac{x+1}{\sqrt[3]{x}+2}=-\lim\limits_{x\to-1}\dfrac{\sqrt[3]{x}+2}{\sqrt[3]{x}+2}(x^{\frac{2}{3}}-2x^{\frac{1}{3}}+4)$

$=\underline{\underline{-12}}$

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7) $\sqrt{2+3x}-\sqrt{2-x}=\dfrac{2+3x-(2-x)}{\sqrt{2+3x}+\sqrt{2-x}}=\dfrac{4x}{\sqrt{2+3x}+\sqrt{2-x}}$

$\sqrt{1-\cos x}=\sqrt{1-(1-2\sin^2\frac{x}{2})}=\sqrt{2}\,|\sin\frac{x}{2}|\xrightarrow[x<0]{}-\sqrt{2}\sin\frac{x}{2}$

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$\lim\limits_{x\to0^-}\left(\dfrac{4}{\sqrt{2+3x}+\sqrt{2-x}}\right)\cdot\dfrac{1}{-\sqrt{2}}\cdot\dfrac{2\cdot\frac{x}{2}}{\sin\frac{x}{2}}=\dfrac{4}{2\sqrt{2}}\cdot\dfrac{1}{-\sqrt{2}}\cdot2=\underline{\underline{-2}}$

8) $\lim\limits_{x\to0}\dfrac{k+\cos(\sqrt{a}x)}{kx^2}=3\longrightarrow k+\cos(0)=0\Rightarrow k=-1$

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$\lim\limits_{x\to0}\dfrac{1-\cos(\sqrt{a}x)}{x^2}=\lim\limits_{x\to0}\dfrac{2\sin^2(\frac{\sqrt{a}}{2}x)}{\frac{4}{a}\cdot\frac{a}{4}x^2}=\dfrac{2}{\frac{4}{a}}=\dfrac{a}{2}=3\Rightarrow a=6$

$\dfrac{a}{k}=\underline{\underline{-6}}$

9) $\lim\limits_{x\to3a^+}\dfrac{2\sqrt{x-3a}+3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\cdot\sqrt{x+3a}}=\lim\limits_{x\to3a^+}\dfrac{2+3\left(\frac{\sqrt{x}-\sqrt{3a}}{\sqrt{x-3a}}\right)}{\sqrt{x+3a}}=L$

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$\lim\limits_{x\to3a^+}\dfrac{\sqrt{x}-\sqrt{3a}}{\sqrt{x-3a}}\cdot\dfrac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\times\dfrac{\sqrt{x-3a}}{\sqrt{x-3a}}=\lim\limits_{x\to3a^+}\dfrac{x-3a}{x-3a}\cdot\dfrac{\sqrt{x-3a}}{\sqrt{x}+\sqrt{3a}}=0$

$L=\dfrac{2+3(0)}{\sqrt{6a}}=\dfrac{2}{\sqrt{6a}}=\dfrac{2}{\sqrt{6}}a^{-\frac{1}{2}}=\underline{\underline{\dfrac{\sqrt{6}}{3}a^{-\frac{1}{2}}}}$

10) $x>-1:\ x^2-1<0\to-k[x]+1>0:\ k>-1$
$x<-1:\ x^2-1>0\to-k[x]+1<0:\ k<-\frac{1}{2}$
$\Big\}\ k\in(-1,-\frac{1}{2})$

$k\pi\in(-\pi,-\frac{\pi}{2})$: ناحیه ۳

نشانگر ← $\cos<0$, $k<0$ ← t ناحیه ۳ مثبت

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