شماره ۲۰

رادمهر مختاری

1) $\lim\limits_{x\to 2} \frac{2x-5}{x^2+ax+b} = -\infty$

$\lim\limits_{x\to 2} 2x-5 = -1<0$

$\lim\limits_{x\to 2} x^2+ax+b = 0$ باید در همسایگی ۲ مثبت باشد

$\Rightarrow x^2+ax+b = (x-2)^2 = x^2-4x+4$

$\Rightarrow a+b=0$

2) $\lim\limits_{x\to \sqrt[3]{4}} \frac{x}{x^2+ax+b}$

$\lim\limits_{x\to \sqrt[3]{4}} (x) > 0$

$x^2+ax+b = f(x)$

$\lim\limits_{x\to\sqrt[3]{4}} f(x) = 0$ در همسایگی مثبت است

$\Rightarrow f(x) = (x-\sqrt[3]{4})^2 = x^2 - 2\sqrt[3]{4}x + 2\sqrt[3]{2}$

$\left[\frac{b}{a}\right] = \left[\frac{+2\sqrt[3]{2}}{-2\sqrt[3]{4}}\right] = \left[-\sqrt[3]{\frac{1}{2}}\right] = \underline{\underline{-1}}$

3) $\lim\limits_{x\to(\frac{\sqrt{3}}{3})^+} \frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = \lim \frac{a-1}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = -\infty$

$\Rightarrow \left|\frac{\sqrt{3}}{3}\times\frac{\sqrt{3}}{3}+a\right|+a=0 \longrightarrow -a = \left|a+\frac{1}{3}\right| \Rightarrow a^2 = a^2+\frac{2}{3}a+\frac{1}{9} \Rightarrow a = -\frac{1}{6}$

$\hookrightarrow a<0$

$[a] = \underline{\underline{-1}}$

4) $x > -\frac{1}{2} \longrightarrow x^2 < \frac{1}{4} \to \frac{1}{x^2} > 4 \to \frac{-2}{x^2} < -8 \to \left[\frac{-2}{x^2}\right]_{x\to -\frac{1}{2}} = \underline{\underline{-9}}$

$\hookrightarrow \frac{3}{x^2} > 12 \longrightarrow \left[\frac{3}{x^2}\right] = \underline{\underline{12}}$

$\Rightarrow \lim\limits_{x\to -\frac{1}{2}^+} \frac{16x-(-9)}{24x+12} = \lim\limits_{x\to-\frac{1}{2}^+} \frac{16x+9}{24(x+\frac{1}{2})} = +\infty$

5) $x>2 \longrightarrow x^3 > 8 \longrightarrow [x^3] = 8$

$\lim\limits_{x\to 2} \frac{x^2-4}{x^3-(8)} = \lim\limits_{x\to 2} \frac{x+2}{x^2+2x+4} = \frac{4}{12} = \underline{\underline{\frac{1}{3}}}$

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6) $\lim\limits_{x\to -8} \dfrac{\overbrace{(x+2)}^{-6}(x+8)}{6(\sqrt[3]{x}+2)} = -\lim\limits_{x\to -8} \dfrac{x+8}{\sqrt[3]{x}+2} = -\lim\limits_{x\to -8} \dfrac{\sqrt[3]{x}+2}{\sqrt[3]{x}+2}(x^{\frac{2}{3}} - 2x^{\frac{1}{3}} + 4)$

$= \underline{\underline{-12}}$

7) $\sqrt{2+3x} - \sqrt{2-x} = \dfrac{2+3x-(2-x)}{\sqrt{2+3x}+\sqrt{2-x}} = \dfrac{4x}{\sqrt{2+3x}+\sqrt{2-x}}$

$\sqrt{1-\cos x} = \sqrt{1-(1-2\sin^2\frac{x}{2})} = \sqrt{2}\,|\sin\frac{x}{2}| \xrightarrow[x<0]{} -\sqrt{2}\sin\frac{x}{2}$

$\lim\limits_{x\to 0^-} \left(\dfrac{4}{\sqrt{2+3x}+\sqrt{2-x}}\right)\cdot\dfrac{1}{-\sqrt{2}}\cdot\dfrac{2\cdot\frac{x}{2}}{\sin\frac{x}{2}} = \dfrac{4}{2\sqrt{2}}\cdot\dfrac{1}{-\sqrt{2}}\cdot 2 = \underline{\underline{-2}}$

8) $\lim\limits_{x\to 0} \dfrac{k+\cos(\sqrt{a}x)}{kx^2} = 3 \longrightarrow k+\cos(0)=0 \Rightarrow k=-1$

$\lim\limits_{x\to 0} \dfrac{1-\cos(\sqrt{a}x)}{x^2} = \lim\limits_{x\to 0} \dfrac{2\sin^2(\frac{\sqrt{a}}{2}x)}{\frac{4}{a}\cdot\frac{a}{4}x^2} = \dfrac{2}{\frac{4}{a}} = \dfrac{a}{2} = 3 \Rightarrow a=6$

$\dfrac{a}{k} = \underline{\underline{-6}}$

9) $\lim\limits_{x\to 3a^+} \dfrac{2\sqrt{x-3a}+3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\cdot\sqrt{x+3a}} = \lim\limits_{x\to 3a^+} \dfrac{2+3\left(\frac{\sqrt{x}-\sqrt{3a}}{\sqrt{x-3a}}\right)}{\sqrt{x+3a}} = L$

$\lim\limits_{x\to 3a^+} \dfrac{\sqrt{x}-\sqrt{3a}}{\sqrt{x-3a}}\cdot\dfrac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\times\dfrac{\sqrt{x-3a}}{\sqrt{x-3a}} = \lim\limits_{x\to 3a^+} \dfrac{x-3a}{x-3a}\cdot\dfrac{\sqrt{x-3a}}{\sqrt{x}+\sqrt{3a}} = 0$

$L = \dfrac{2+3(0)}{\sqrt{6a}} = \dfrac{2}{\sqrt{6a}} = \dfrac{2}{\sqrt{6}}a^{-\frac{1}{2}} = \underline{\underline{\dfrac{\sqrt{6}}{3}a^{-\frac{1}{2}}}}$

10) $x>-1:\ x^2-1<0 \to -k[x]+1>0:\ k>-1$

$x<-1:\ x^2-1>0 \to -k[x]+1<0:\ k<-\frac{1}{2}$ $\Big\}\ k\in(-1,-\frac{1}{2})$

$k\pi\in(-\pi,-\frac{\pi}{2})$: ناحیه ۳

نتایج ← $\cos<0$, $k<0$ ← ناحیه ۳ مثلثات