

۱) $(x^2 - 2)^2 \leq x^2 - 1 \Rightarrow x^4 - 4x^2 + 4 \leq x^2 - 1 \Rightarrow x^4 - 5x^2 + 5 \leq 0$
 $(x^2 - 1)(x^2 - 4) \leq 0 \Rightarrow (x-1)(x+1)(x-2)(x+2) \leq 0$
 $x \in (-2, -1] \cup [1, 2)$

۲) $(\sqrt{x})^2 + a(\sqrt{x}) + b \leq 0$

$(x - \sqrt{x})^2 \leq x \Rightarrow x^2 - 2\sqrt{x}x + x \leq x \Rightarrow x^2 - 2\sqrt{x}x \leq 0$
 $x(x - 2\sqrt{x}) \leq 0 \Rightarrow x \leq 2\sqrt{x} \Rightarrow \sqrt{x} \leq 2 \Rightarrow x \leq 4$
 $x \in [0, 4]$

۳) $\lim_{n \rightarrow \infty} \frac{a-1}{\left| \frac{\sqrt{n}}{n}n + a \right| + a} = \lim_{n \rightarrow \infty} \frac{a-1}{|a + \frac{1}{\sqrt{n}}| + a}$

$|a + \frac{1}{\sqrt{n}}| \leq a + \frac{1}{\sqrt{n}}$
 $a + \frac{1}{\sqrt{n}} \rightarrow a$
 $[a]_{n \rightarrow \infty} = a$

۴) $x > -\frac{1}{x} \Rightarrow x^2 < \frac{1}{x} \Rightarrow \frac{1}{x^2} > x \Rightarrow \frac{x}{x^2} > x \Rightarrow \left[\frac{x}{x^2} \right] = 1$

$\Rightarrow -\frac{x}{x^2} < -1 \Rightarrow \left[-\frac{x}{x^2} \right] = -1$

$\lim_{n \rightarrow (-\frac{1}{x})^+} \frac{14n+9}{14n+12} = \frac{-1+9}{-1+12} = \frac{8}{11}$

۵) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - 1} = \frac{0}{0}$

$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - 1} = \frac{(x-1)(x+1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{x+1}{x+1} = \frac{2}{2} = 1$

$$n \rightarrow -1^- \text{ s) } \frac{(n+2)(n+1)}{4(2+\sqrt{n})} \text{ s } \frac{0^-}{0^-}$$

120

$$\lim_{n \rightarrow -1} \frac{(n+2)(n+1)}{4(2+\sqrt{n})} \times \frac{\sqrt[3]{n^2} - \sqrt[3]{n} + \epsilon}{\sqrt[3]{n^2} - \sqrt[3]{n} + \epsilon} = \lim_{n \rightarrow -1} \frac{(-1+2)(-1+1) \times 12}{4(-1+1)} = \boxed{-12}$$

6

$$\lim_{n \rightarrow 0} \frac{\sqrt{2+\sqrt{n}} - \sqrt{2-n}}{\sqrt{1-\cos n}} \times \frac{\sqrt{2+\sqrt{n}} + \sqrt{2-n}}{\sqrt{2+\sqrt{n}} + \sqrt{2-n}} \times \frac{\sqrt{1+\cos n}}{\sqrt{1+\cos n}} =$$

0

7

$$\lim_{n \rightarrow 0} \frac{n \times \sqrt{2}}{\sqrt{1-\cos n}} \times \frac{1}{\sqrt{2}} = \lim_{n \rightarrow 0} \frac{n \sqrt{2}}{|\sin n| \sqrt{2}} = 2 \times \frac{n}{-\sin n} = \boxed{-2}$$

$$Hofst) - \frac{\sqrt{k} \cdot (\sqrt{a_n})}{\sqrt{k a_n}} \text{ s } 2$$

چون منجم برابر صفر است و حاصل حد های صحیح و غیر صفر است پس حد صفر است نیز برابر صفر است!

$$\lim_{n \rightarrow 0} k + \cos(\sqrt{a_n}) = 0 \rightarrow k+1=0 \rightarrow \boxed{k=-1}$$

8

$$\lim_{n \rightarrow 0} \frac{-1 + \cos \sqrt{a_n}}{-n^2} \sim \lim_{n \rightarrow 0} \frac{-1 + 1 - \frac{(\sqrt{a_n})^2}{2}}{-n^2} = \frac{-\frac{a_n}{2}}{-n^2} = \frac{a}{2}$$

$$\frac{a}{2} = 2 \rightarrow \boxed{a=4} \rightsquigarrow \frac{a}{k} = \boxed{-4}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n} \sqrt{a_n}}{\sqrt{a_n} \sqrt{a_n} \sqrt{a_n}} + \frac{\sqrt[3]{n} (\sqrt{n} - \sqrt{a_n})}{\sqrt{n-a_n} \sqrt{n+a_n}} = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt{a_n}} + \frac{\sqrt[3]{n} (\sqrt{n} - \sqrt{a_n})}{\sqrt{n-a_n} \sqrt{n+a_n}} \times \frac{\sqrt{n} + \sqrt{a_n}}{\sqrt{n} + \sqrt{a_n}}$$

9

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt{a_n}} + \frac{\sqrt[3]{n} (\sqrt{n} - \sqrt{a_n})}{\sqrt{a_n} \sqrt{n+a_n} (\sqrt{n} + \sqrt{a_n})} = \lim_{n \rightarrow \infty} \left(\frac{\sqrt[3]{n}}{\sqrt{a_n}} + \frac{\sqrt[3]{n} \sqrt{a_n}}{\sqrt{a_n} (\sqrt{n} + \sqrt{a_n})} \right) = \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n}}{\sqrt{a_n}} + \dots \rightsquigarrow \frac{\sqrt[3]{n}}{\sqrt{a_n}} = \sqrt{\frac{n}{a_n}} = \sqrt{\frac{n}{n^2}} = \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow -1^+} \frac{1+k}{2-1} \text{ s } -\infty \text{ s) } k+1 > 0 \left\{ \begin{array}{l} \text{s) } \rightarrow k > -1 \\ \text{f) } \rightarrow k < -\frac{1}{2} \end{array} \right.$$

$$\lim_{n \rightarrow -1^-} \frac{1+k}{2-1} = -\infty \quad 1+k < 0 \rightarrow k < -\frac{1}{2}$$

10

$$\lim_{x \rightarrow -1} \frac{1 - k[x]}{x^2 - 1} = -\infty$$

$\rightarrow \lim_{x \rightarrow (-1)^+} \frac{1 - k(-1)}{0^-} = -\infty \rightarrow 1 + k > 0 \rightarrow k > -1$

$\rightarrow \lim_{x \rightarrow (-1)^-} \frac{1 - k(-1)}{0^+} = -\infty \rightarrow 1 + k < 0 \rightarrow k < -1$

$\rightarrow -1 < k < -\frac{1}{r} \quad \begin{cases} -\pi < k\pi < -\frac{\pi}{r} \\ -1 < 0 < k\pi < 0 \end{cases}$



مولفه های اول و دوم هر دو منفی هستند پس در ناحیه ی سوم است!