ماهان نژاداکبر

تکلیف ۳

۱۲ـB

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$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=-\infty$$

چون حاصل حد است بی‌نهایت شده، پس مخرج وقتی $x=2$ ← صفر می‌شود و باید ریشه مضاعف باشد زیرا که حد است و چپ و راست برابر بوده، تغییر علامت نداده‌اند:

$$(x^2+ax+b)=k(x-2)^2=k(x^2-4x+4)\Rightarrow k=1 \quad b=4 \quad a=-4$$

$$\lim_{x\to 2}\frac{2x-5}{x^2-4x+4}=\frac{2x-5}{(x-2)^2}=\frac{-1}{0^+}=-\infty \qquad a+b=-4+4=0$$

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$$\lim_{x\to\sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \quad \begin{cases}\lim\limits_{x\to(\sqrt[3]{4})^+} x^2+ax+b=0^+ \\ \lim\limits_{x\to(\sqrt[3]{4})^-} x^2+ax+b=0^+\end{cases}$$

$$\Rightarrow x^2+ax+b=(x-\sqrt[3]{4})^2=x^2+\sqrt[3]{16}-2\sqrt[3]{4}x\Rightarrow\begin{cases}a=-\sqrt[3]{32}\\ b=\sqrt[3]{16}\end{cases}$$

$$\Rightarrow\left[\frac{b}{a}\right]=\left[\frac{\sqrt[3]{16}}{-\sqrt[3]{32}}\right]=\left[-\sqrt[3]{\frac{4}{2}}\right]\overset{I}{=}-2$$

$$1<\frac{4}{2}<8\Rightarrow 1<\sqrt[3]{\frac{4}{2}}<2\Rightarrow -1>-\sqrt[3]{\frac{4}{2}}>-2$$

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$$\lim_{x\to(\frac{1}{\sqrt{2}})^+}\frac{[-x]+a}{\left|\frac{\sqrt{2}}{2}x-a\right|+a}=-\infty$$

$$x>\frac{\sqrt{2}}{2}\Rightarrow -x<-\frac{\sqrt{2}}{2}\Rightarrow[-x]=-1$$

$$\Rightarrow\lim_{x\to(\frac{1}{\sqrt{2}})^+}\frac{a-1}{\left|\frac{\sqrt{2}}{2}x-a\right|+a}=-\infty\begin{cases}\frac{a-1}{-\frac{1}{2}}=-\infty \quad a<-\frac{1}{2} & \text{غ ق ق}\\ \lim\frac{a-1}{2a+\frac{\sqrt{2}}{2}x}=-\infty \quad a>-\frac{1}{2}\end{cases}$$

$$\rightarrow \lim_{x\to \frac{\sqrt{2}}{2}^{+}} \left(2x + \frac{\sqrt{2}}{2}x\right) = 0 \rightarrow 2x + \frac{1}{2} = 0 \rightarrow x = -\frac{1}{4}$$

$$\rightarrow [x] + \left[-\frac{1}{4}\right] = -1$$

ج -

$$\lim_{x\to (-\frac{1}{2})^{+}} \frac{16x - \left[-\frac{3}{25}\right]}{24x + \left[\frac{5}{25}\right]} = \frac{16x - (-9)}{24x + 12}$$

$$x > -\frac{1}{2} \rightarrow -x < \frac{1}{2} \rightarrow \frac{1}{25} > 4 \quad \begin{cases} \frac{3}{25} > 12 \rightarrow \left[\frac{3}{25}\right] = 12 \\ -\frac{3}{25} < -x \rightarrow \left[-\frac{5}{25}\right] = -9 \end{cases}$$

$$\Rightarrow \lim_{x\to (-\frac{1}{2})^{+}} \frac{16x + 9}{24x + 12} = \frac{-8+9}{0^{+}} = +\infty$$

$$x > -\frac{1}{2} \rightarrow 24x > -12 \rightarrow 24x + 12 > 0$$

د -

$$\lim_{x\to 2^{+}} \frac{x^2 - 4}{x^3 - [x^3]} = \frac{x^2-4}{x^3-8} = \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \frac{x+2}{x^2+2x+4} = \frac{4}{12} = \frac{1}{3}$$

$$\boxed{x > 2 \rightarrow x^3 > 8 \rightarrow [x^3] = 8}$$

ه -

$$\lim_{x\to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{0}{0}$$

$$\lim_{x\to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{(x+8)(x+2)}{6(\sqrt[3]{x}+2)} = \frac{(\sqrt[3]{x}+2)(\sqrt[3]{x^2}+4-2\sqrt[3]{x})(x+2)}{6(\sqrt[3]{x}+2)}$$

$$= \frac{(\sqrt[3]{x^2}+4-2\sqrt[3]{x})(x+2)}{6} = \frac{(4+4+4)(-6)}{6} = \frac{12\times(-6)}{6} = -12$$

و -

$$\lim_{x\to 0^{-}} \frac{\sqrt{2+3x} - \sqrt{2-x}}{\sqrt{1-\cos x}} = \frac{\sqrt{2+3x} - \sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x} + \sqrt{2-x}}{\sqrt{2+3x} + \sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0^-} \frac{4x}{\sqrt{1-\cos^2 x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+x^2}+\sqrt{1-x}} = \lim_{x\to 0^-} \frac{4x}{|\sin x|} \times \frac{\sqrt{2}}{2\sqrt{2}}$$

$x<0 \to \sin x<0 \to |\sin x| = -\sin x$

$$= \lim_{x\to 0^-} \frac{4x}{-\sin x} \times \frac{1}{2} = -4 \times \frac{1}{2} = -2$$

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$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = \frac{k+\left(1-\frac{ax^2}{2}\right)}{kx^2} = \frac{k+1-\frac{ax^2}{2}}{kx^2} = 3$$

$$\overset{\text{Hop}}{\Longrightarrow} \lim_{x\to 0} \frac{-ax}{2kx} = \frac{-a}{2k} = 3 \Rightarrow \frac{a}{k} = -6$$

۹ـ

$$\lim_{x\to (2a)^+} \frac{2\sqrt{x-2a}+3\sqrt{x}-\sqrt{2}\sqrt{a}}{\sqrt{x^2-4a^2}} = \frac{2\sqrt{x-2a}}{\sqrt{x^2-4a^2}} + \frac{3(\sqrt{x}-\sqrt{2a})}{\sqrt{x^2-4a^2}}$$

$$= \frac{2}{\sqrt{x+2a}} + \frac{3(\sqrt{x}-\sqrt{2a})}{\sqrt{x^2-4a^2}} \times \frac{\sqrt{x}+\sqrt{2a}}{\sqrt{x}+\sqrt{2a}} = \frac{2}{\sqrt{4a}} + \frac{3(\sqrt{x-2a})}{\sqrt{x+2a}}$$

$$= \boxed{\frac{2}{\sqrt{4a}}}$$

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$$\lim_{x\to -1} \frac{1-k[x]}{x^2-1} = \begin{cases} \lim_{x\to -1^+} \frac{1+k}{x^2-1} = -\infty \\ \lim_{x\to -1^-} \frac{1+2k}{x^2-1} = -\infty \end{cases}$$

$$\Rightarrow \frac{1+k}{0^-} = -\infty \to 1+k>0 \Rightarrow k>-1$$

$$\Rightarrow \frac{1+2k}{0^+} = -\infty \to 1+2k<0 \to k<-\frac{1}{2}$$

$$-1<k<-\frac{1}{2}$$

ناحیه سوم

$\Rightarrow -\pi<k\pi<-\frac{\pi}{2} \Rightarrow$ صعودی $\cos(-\pi)<\cos k\pi<\cos(-\frac{\pi}{2})$

$$-1<\cos k\pi<0$$