امیر کاشف ۱۸,۵

① $\lim_{x\to 2} \frac{2x-5}{x^2+ax+b} = -\infty \quad a+b\ ?$ ۲

$x=2$

$\frac{-1}{4+2a+b} = -\infty \to 0 \Rightarrow (x-2)^2 = x^2+ax+b \Rightarrow b=4,\ a=-4$

$\Rightarrow a+b=0$

② $\lim_{x\to(\sqrt[3]{4})} \frac{x}{x^2+ax+b} = +\infty \xrightarrow{x=\sqrt[3]{4}} \frac{\sqrt[3]{4}}{0} = +\infty \Rightarrow x^2+ax+b = (x-\sqrt[3]{4})^2 \Rightarrow$ ۲

$\left[\frac{b}{a}\right]$

$b = 2^{\frac{4}{3}}$, $a = -2\sqrt[3]{4}$

$\Rightarrow \left[\frac{b}{a}\right] = \left[\frac{2^{\frac{4}{3}}}{-2\times 2^{\frac{2}{3}}}\right] = -1$

③ $\lim_{x\to\left(\frac{1}{\sqrt{3}}\right)^+} \frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = -\infty \quad [a]$ ۲

$x=\frac{1}{\sqrt{3}}$

$\frac{}{\left|\frac{1}{3}+a\right|+a} \to 0$

$a<-\frac{1}{3} \quad \frac{}{-\frac{1}{3}}$ ✗

$a>-\frac{1}{3} \quad \frac{}{2a+\frac{1}{3}}$ ✓ $\Rightarrow 2a+\frac{1}{3}=0 \Rightarrow a=-\frac{1}{6}$

$[a]=-1$

④ $\lim_{x\to\left(-\frac{1}{2}\right)^+} \frac{14x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} = \frac{14x+9}{\cancel{24x}+12} \overset{\frac{0}{0}}{\underset{Hop}{=}} \frac{14}{\cancel{24}} = \frac{2}{\cancel{3}} \quad +\infty$ ۱,۵

$12x$

$\underbrace{\left[\frac{3}{x^2}\right]}_{12}$

$x\to-\frac{1}{2}^+ \Rightarrow x^2 \to \frac{1}{4}^- \Rightarrow \frac{1}{x^2} \to 4^+$

⑤ $\lim_{x\to 2^+} \frac{x^2-4}{x^3-[x^3]} = \frac{x^2-4}{x^3-8} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)} = \frac{4}{12} = \boxed{\frac{1}{3}}$ ۲

⑥ $\lim_{x\to-8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{0}{0} \overset{Hop}{\Longrightarrow} \frac{2x+10}{6\frac{1}{3\sqrt[3]{x^2}}} = \frac{-16+10}{\frac{6}{3\sqrt[3]{64}}} = \frac{-6\times 12}{\cancel{6}} = -12$ ۲

⑦ $\lim_{x\to 0^-} \frac{\sqrt{2+x^2}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+x^2}+\sqrt{2-x}}{\sqrt{2+x^2}+\sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} = \frac{x\times\sqrt{2}}{2\sqrt{2}\left(\sqrt{1-\cos^2 x}\right)} = \frac{2x}{-\sin x} = -2$ ۲

$\sqrt{2+x^2}+\sqrt{2-x} \to 2\sqrt{2}$, $\sqrt{1+\cos x} \to \sqrt{2}$

$\sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = |\sin x| = -\sin x$

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}x)}{kx^2}=3$$

۸

$$\underbrace{\ }_{0}\longrightarrow k+\cancel{\cos(\sqrt{a}\,x)}_{1}=0\Rightarrow \boxed{k=-1}$$

$$\lim_{x\to 0}\frac{-1+\cos(\sqrt{a}\,x)}{-x^2}=3\ \xrightarrow[\text{Hop}]{\frac{0}{0}}\ \frac{\sqrt{a}\,(\cancel{-}\sin(\sqrt{a}x))}{\cancel{2x}}=3\Rightarrow\frac{a}{2}=3\Rightarrow\boxed{a=6}$$

$$\Rightarrow\frac{a}{k}=-6$$

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$$\lim_{x\to(3a)^+}\frac{\overbrace{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}^{0}}{\underbrace{\sqrt{x^2-9a^2}}_{0}}$$

۹

$$\xrightarrow{\text{Hop}}\frac{\cancel{2}\frac{1}{\cancel{2}\sqrt{x-3a}}+3\frac{1}{2\sqrt{x}}}{\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-9a^2}}}=\left(\frac{1}{\sqrt{x-3a}}+\frac{3}{2\sqrt{x}}\right)\frac{\cancel{\sqrt{x-3a}}\sqrt{x+3a}}{x}=$$

$$\sqrt{x-3a}\sqrt{x+3a}$$

$$\frac{2\sqrt{x}+3\sqrt{x-3a}}{\sqrt{x-3a}\;2\sqrt{x}}$$

۲

$$\frac{(2x+3\sqrt{x-3a})\sqrt{x+3a}}{x\times 2\sqrt{x}}$$

$$\lim_{x\to(-1)}\frac{1-k[x]}{x^2-1}=-\infty\qquad (k\pi,\cos k\pi)$$

۱۰

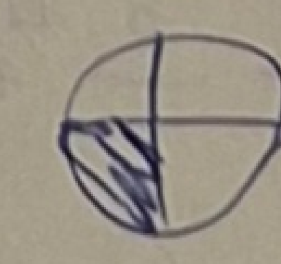
$$-1^+:\ \frac{1+k}{0^-}=-\infty\Rightarrow 1+k>0\Rightarrow k>-1$$

$$-1^-:\ \frac{1+2k}{0^+}=-\infty\Rightarrow 1+2k<0\Rightarrow 2k<-1\Rightarrow \boxed{k<-\tfrac{1}{2}}$$

$$\Rightarrow -1<k<-\tfrac{1}{2}\Longrightarrow (k\pi,\cos k\pi)$$

ناحیه سوم

۲



۴

$$x > -\frac{۱}{۲} \to x^{۲} < \frac{۱}{۴} \to \frac{۱}{x^{۲}} > ۴ \to \frac{۳}{x^{۲}} > ۱۲ \leadsto \left[\frac{۳}{x^{۲}}\right] = ۱۲$$

$$\hookrightarrow -\frac{۲}{x^{۲}} < -۸ \leadsto \left[-\frac{۲}{x^{۲}}\right] = -۹$$

$$\lim_{x \to (-\frac{۱}{۲})^{+}} \frac{۱۶x + ۹}{۲۴x + ۱۲} = \frac{-۸ + ۹}{-۱۲^{+} + ۱۲} = \frac{۱}{۰^{+}} = \boxed{+\infty}$$

۹

$$\lim_{x \to ۳a^{+}} \frac{۲\sqrt{\cancel{x-۳a}}}{\sqrt{\cancel{x-۳a}}\sqrt{x+۳a}} + \frac{۳(\sqrt{x} - \sqrt{۳a})}{\sqrt{x-۳a}\sqrt{x+۳a}} = \lim_{x \to ۳a^{+}} \frac{۲}{\sqrt{۶a}} + \frac{۳(\sqrt{x} - \sqrt{۳a})}{\sqrt{x-۳a}\sqrt{x+۳a}} \times \frac{\sqrt{x} + \sqrt{۳a}}{\sqrt{x} + \sqrt{۳a}}$$

$$\lim_{x \to ۳a^{+}} \frac{۲}{\sqrt{۶a}} + \frac{۳(\cancel{x-۳a})^{\sqrt{x-۳a}}}{\sqrt{\cancel{x-۳a}}\sqrt{x+۳a}(\sqrt{x} + \sqrt{۳a})} = \lim_{x \to ۳a^{+}} \left(\frac{۲}{\sqrt{۶a}} + \frac{۳\sqrt{\cancel{x-۳a}}^{\,۰}}{\sqrt{\cancel{x-۳a}}(\sqrt{x}+\sqrt{۳a})}\right) = \lim_{x \to ۳a^{+}} \frac{۲}{\sqrt{۶a}} + ۰ \leadsto \frac{۲}{\sqrt{۶a}} = \sqrt{\frac{۴}{۶a}} = \sqrt{\frac{۲}{۳a}}$$