امیر کاشف

① $\lim_{x\to 2} \frac{2x-5}{x^2+ax+b} = -\infty$ $\quad a+b$ ؟

$x=2$

$\frac{-1}{\underbrace{x^2+ax+b}_{\to 0}} = -\infty \Rightarrow (x-2)^2 = x^2+ax+b \Rightarrow b=4$ و $a=-4$

$\Rightarrow a+b=0$

---

② $\lim_{x\to(\sqrt[3]{4})} \frac{x}{x^2+ax+b} = +\infty \xrightarrow{x=\sqrt[3]{4}} \frac{\sqrt[3]{4}^{+}}{0} = +\infty \Rightarrow x^2+ax+b = (x-\sqrt[3]{4})^2 \Rightarrow$

$\left[\frac{b}{a}\right]$ $\qquad \boxed{b = 2^{\frac{4}{3}}}$ $\boxed{a = -2\sqrt[3]{4}}$

$\Rightarrow \left[\frac{b}{a}\right] = \left[\frac{2^{\frac{4}{3}}}{-2\times 2^{\frac{2}{3}}}\right] = -1$

---

③ $\lim_{x\to(\frac{1}{\sqrt3})^+} \frac{[-x]+a}{|\frac{\sqrt3}{3}x+a|+a} = -\infty$ $\quad [a]$

$x=\frac{1}{\sqrt3}$

$\frac{\quad}{\underbrace{|\frac13+a|+a}_{0}}$ $\quad a<-\frac13 \Rightarrow \frac{1}{3}$ ✗ $\qquad [a]=-1$

$a>-\frac13 \quad 2a+\frac13$ ✓ $\Rightarrow 2a+\frac13=0 \Rightarrow a=-\frac16$

---

④ $\lim_{x\to(-\frac12^+)} \frac{16x-[-\frac{2}{x^2}]}{24x+\underbrace{[\frac{3}{x^2}]}_{12}} = \frac{16x+9}{24x+12} \overset{\frac00}{\underset{Hop}{=}} \frac{16}{24} = \frac{2}{3}$

$x\to-\frac12^+ \Rightarrow x^2 \to \frac14^- \Rightarrow \frac{1}{x^2}\to 4^+$

---

⑤ $\lim_{x\to2^+} \frac{x^2-4}{x^3-[x^3]} = \frac{x^2-4}{x^3-8} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+4+2x)} = \frac{4}{12} = \boxed{\frac{1}{3}}$

---

⑥ $\lim_{x\to-8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac00 \overset{Hop}{\Longrightarrow} \frac{2x+10}{6\frac{1}{3\sqrt[3]{x^2}}} = \frac{-16+10}{\frac{6}{3\sqrt[3]{64}}} = \frac{-6\times\cancel{12}^{2}}{\cancel{6}} = -12$

---

⑦ $\lim_{x\to0^-} \frac{\overbrace{\sqrt{2+3x}-\sqrt{2-x}}^{4x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\underbrace{\sqrt{2+3x}+\sqrt{2-x}}_{2\sqrt2}} \times \frac{\overbrace{\sqrt{1+\cos x}}^{\sqrt2}}{\sqrt{1+\cos x}} = \frac{4x\times\cancel{\sqrt2}}{2\cancel{\sqrt2}\,(\underbrace{\sqrt{1-\cos^2 x}}_{\sin^2 x})} = \frac{2x}{\underbrace{-\sin x}_{1}} = -2$

$\cancel{|\sin x|}$

$-\sin x$

۸

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}\,x)}{kx^2}=3$$

$$\underbrace{\ }_{0}\longrightarrow k+\cancel{\cos(\sqrt{a}\,x)}_{1}=0\Rightarrow \boxed{k=-1}$$

$$\lim_{x\to 0}\frac{-1+\cos(\sqrt{a}\,x)}{-x^2}=3\ \xrightarrow[\text{Hop}]{\frac{0}{0}}\ \frac{\sqrt{a}\,(-\sin(\sqrt{a}\,x))}{-2x}=3\Rightarrow\frac{a}{2}=3\Rightarrow\boxed{a=6}$$

$$\Rightarrow\frac{a}{k}=-6$$

۹

$$\lim_{x\to(3a)^+}\frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\underbrace{\sqrt{x^2-9a^2}}_{0}}$$

$$\xRightarrow{\text{Hop}}\frac{2\frac{1}{2\sqrt{x-3a}}+3\frac{1}{2\sqrt{x}}}{\frac{2x}{2\sqrt{x^2-9a^2}}}=\left(\frac{1}{\sqrt{x-3a}}+\frac{3}{2\sqrt{x}}\right)\frac{\sqrt{x-3a}\sqrt{x+3a}}{x}$$

$$\frac{2\sqrt{x}+3\sqrt{x-3a}}{\sqrt{x-3a}\;2\sqrt{x}}$$

$$\frac{(2x+3\sqrt{x-3a})\sqrt{x+3a}}{x\times 2\sqrt{x}}$$

۱۰

$$\lim_{x\to(-1)}\frac{1-k[x]}{x^2-1}=-\infty\qquad(k\pi,\cos k\pi)$$

$$-1^+:\ \frac{1+k}{0^-}=-\infty\Rightarrow 1+k>0\Rightarrow k>-1$$

$$-1^-:\ \frac{1+2k}{0^+}=-\infty\Rightarrow 1+2k<0\Rightarrow 2k<-1\Rightarrow k<-\frac{1}{2}$$

$$\Rightarrow -1<k<-\frac{1}{2}\Longrightarrow(k\pi,\cos k\pi)$$

ناحیه سوم