۱-

$\Delta = 0 \to (m+3)^2 - 4(\frac{m}{2})(6) \to m^2 - 6m + 9 = 0 \to m = 3 \to a = \frac{1}{2}$

$\frac{\sqrt{6(x+\frac{1}{2})^2}}{2|x^3+\frac{1}{8}|} = \frac{\sqrt{6}\,|\cancel{x+\frac{1}{2}}|}{2|\cancel{x+\frac{1}{2}}||x^2-\frac{1}{2}x+\frac{1}{4}|} = \frac{\sqrt{6}}{2|\frac{3}{4}|} = \frac{2\sqrt{6}}{3} = \frac{\cancel{2}\tan b \times \cancel{\sqrt{6}}}{\cancel{\sqrt{2}}} = \frac{\cancel{2}\sqrt{2}\times\sqrt{3}}{3} \to b = 30^\circ = \frac{\pi}{6}$

۲-

پیوستگی $f \to x=1 \to \lim_{x\to 1}\frac{f(x)}{x-1} =$ موجود $\to f(1)=0 \to 1+a+b=0$

$5x^2 - ax + b \xrightarrow{x=1} 5 - a + b = 0 \to 6 + 2b = 0 \to b = -3 \to a = 2$

$\left[\frac{-3-2(2)}{2}\right] = \left[-\frac{7}{2}\right] = -4$

۳-

$f(1) = f(1^\pm) \to f(1) = f(1^-) = \tan\frac{3\pi}{4} = -1 = f(1^+) = \frac{|(x+2)(x-1)|}{a(1-x)} = \frac{-x-2}{-a} = \frac{3}{-a} = -1 \to a = 3$

$f(5) = f(5^\pm),\ f(5^-) = \frac{\cancel{4k}-7}{3\times\cancel{4}} = \frac{-7}{3}$

$f(5) = 10b = \frac{-7}{3} + b = \frac{-7}{30} \to ab = 3 \times \frac{-7}{3\times 10} = \frac{-7}{10}$

۴-

$a[x] + a + b[x] + b[a] + b \to f(x) = a + b + b[a] = f(a)$

$x = k \to$ ناپیوستگی $\to a[x] + b[x] = 0 \to a = -b$

$\frac{a[a]}{\cancel{a-a}-a[a]} = -1$

۵-

$f(x) = b[x^2 - ax] - 2a \xrightarrow{\text{پیوستگی}} b = 0 \to f(x) = -2a = f(b) \quad \frac{a}{f(b)} = \frac{a}{-2a} = \frac{-1}{2}$

۶-

$f(a^\pm) = \underbrace{f(a)}_{2} \to \frac{a^2+ma+n}{(a-a)} = \frac{0}{0} \xrightarrow{\text{هوپ}} \frac{2a+m}{-1} = 2 \to 2a + 2 = -m \xrightarrow{*} 2a + 2 = 3a \to a = 2$

صفرهای مخرج ... پس صورت باید صفرشان ۲ باشد

$f(a) = f(2a) = 0 \to (x-2)(x-4) = x^2 - 6x + 8 \quad m = -6,\ 8 = n$

$n - m = 8 - (-6) = 14$

$$f(x)=\begin{cases}(1-a)[x]+(3a^2-1)[-x] & ; x\notin Z\\ b\sin(\frac{\pi}{a}) & x\in Z\end{cases}$$ -۷

$3a^2-1=1-a \to 3a^2+a-2=0 \to a^2+a-6=0$
$(a+3)(a-2)=0 \to a=\frac{-3}{3} , \frac{2}{3}$

$a=-1 \to 2[x]+2[-x]=-2 \quad x\notin Z \to b\sin(-\pi)=0 \to -2\neq 0$ ✗

$a=\frac{2}{3} \to \frac{1}{3}([x]+[-x])=-\frac{1}{3} \quad x\in Z \to b\sin(\frac{3\pi}{2})=-\frac{1}{3} \to b=\frac{1}{3} \to \frac{\frac{2}{3}}{\frac{1}{3}}=2$

$$f(x)=\begin{cases}\frac{\cancel{|x-1|}|x+m+1|}{\cancel{|x-1|}(m|m+1|+1)} & x\neq 1\\ \frac{m}{m+2} & x=1\end{cases}$$ -۸

$f(1^+)=f(1)=\frac{|m+2|}{2m+1}=\frac{m}{m+2}$

$m^2+4m+4-2m^2-m=0$

$m^2-3m-4=0 \quad m=4$ ✓ یا $m=-1$ ✗

$\lim_{x\to 4} f(x)=\frac{21}{24}=\frac{7}{8}$

$a^3+(m-2)a+a^2=0 \xrightarrow[a=-1]{} -\cancel{1}-m+2+\cancel{1} \to m=2$ -۹

$a^2+a=0 \quad a(a+1)=0 \to a=0$ یا $a=-1$

تابع ناصفر ✗

$f(x)=\frac{\sqrt{3}|x+1|}{|x^2+1|}=\frac{\sqrt{3}}{|x^2+x+1|}=\frac{\sqrt{3}}{3}$

$\lim_{x\to 0} f(x)=\frac{\cancel{|x|}(|x|+1)}{\cancel{|x|}(|x|+a)} \to \frac{1}{a}=\frac{2a-1}{3a+2} \to 2a^2-a=3a+2$ -۱۰

$2a^2-3a-2=0$

$a^2-3a-4=0$

$(a-4)(a+1)=0 \to a=\frac{4}{2} , a=-\frac{1}{2}$

$f(x) \to \frac{x^2+|x|}{x^2+2|x|} \to$ ✓ در R پیوسته

$\to \frac{x^2+|x|}{x^2-\frac{|x|}{2}} \to$ در همه‌جا پیوسته نیست ✗

$\lim_{x\to \frac{1}{2}} f(x)=\frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+2(\frac{1}{2})}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}$

نوید لطفی