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تابع f در $x=0$ گوشه داشته و مشتق ندارد
پس در نقطه $x=1$ مشتق دارد

1) $\lim_{x\to 1} f(x) = f(1) \Rightarrow b = a+1$

$$f'(x) = \begin{cases} \frac{2x}{2\sqrt{x^2}} & ; x<1 \\ b\frac{2}{3}x^{-\frac{1}{3}} & ; x\geq 1 \end{cases} \longrightarrow 1 = \frac{2}{3}b \to b = \frac{2}{3} \to a = -\frac{1}{3}$$

$$\to a+b = \frac{1}{3}$$

2) $f'(1) - 2g'(1) = \left(f(x) - 2g(x)\right)'\Big|_{x=1}$

$$h(x) = (f-2g)(x) = \frac{2|x-2|}{\sqrt[3]{x^2}} - 2\,\frac{\sqrt{(x-2)^2}+\sqrt{3x}}{\sqrt[3]{x^2}} = \frac{-2\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$h(x) = -2\sqrt{3}\,x^{\frac{1}{2}-\frac{2}{3}} = -2\sqrt{3}\,x^{\frac{-1}{6}} \to h'(x) = \frac{\sqrt{3}}{3}x^{-\frac{7}{6}} \to h'(1) = \frac{\sqrt{3}}{3}$$

3) $f(x) = \begin{cases} bx+c & ; x<a \\ \frac{1}{x} & ; x\geq a \end{cases}$ ⟶ در $\mathbb{R}$ مشتق‌پذیر ⟶ در $\mathbb{R}$ پیوسته است

$$\lim_{x\to a} f(x) = f(a) \longrightarrow ba+c = \frac{1}{a} \quad \left\{ \begin{array}{l} f'_+(a) = \lim_{x\to a^+} \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \frac{-1}{a^2} \\ f'_-(a) = b \longrightarrow b = \frac{-1}{a^2} \end{array} \right.$$

$$\to \underbrace{\cancel{ba^2}}_{-1} + ac = 1 \to ac = \underline{\underline{+2}}$$

4) $f(x) = \begin{cases} b & ; x<a \\ b+(x-a)^m & ; x\geq a \end{cases} \longrightarrow f'_-(a) = 0$

$$f'_+(a) = \left(m(x-a)^{m-1}\right)\Big|_{x=a}$$

$\to m>1 \to f'_+(a) = 0$

$m<1 \to f'_+(a) =$ نامتناهی × $\longrightarrow m=1 \to f'_+(a) = \underline{\underline{m=1}}$ ✓ فقط

۵) $f \circ g(x)$

$$\underset{x<0}{g(x)} = \frac{1}{x^3 - |x|^3} = \frac{1}{x^3 + x^3} = \frac{1}{2}x^{-3}$$

$$\left.\begin{array}{l} x<0 \to g(x)<0 \\ \quad \hookrightarrow f(x)<0 \end{array}\right\} \qquad \underset{x<0}{f(x)} = \frac{1}{\sqrt[3]{x-|x|}} = \frac{1}{\sqrt[3]{x+x}} = \frac{1}{\sqrt[3]{2}}x^{-\frac{1}{3}}$$

$$h(x) = \underset{\substack{g(x)<0 \\ x<0}}{f \circ g(x)} = \frac{1}{\sqrt[3]{2}}\left(g(x)\right)^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{2}}\left(\frac{1}{2}x^{-3}\right)^{-\frac{1}{3}} = 1 \to h'(-\sqrt[3]{2}) = \underline{\underline{0}}$$

۶) $f'(3) - f(3) = 2$

$$y = -ax - 5 \longrightarrow \left.\begin{array}{l} f(3) = -3a - 5 \\ f'(3) = -a \end{array}\right\} -a + 3a + 5 = 2 \to a = -\frac{3}{2}$$

$$\frac{f(3)}{f'(3)} = \frac{3a+5}{a} = 3 + \frac{5}{a} = 3 - \frac{10}{3} = \underline{\underline{-\frac{1}{3}}}$$

۷) $$g\left(\frac{3}{\sqrt{8}}\right) = 2 \to \underset{\underline{2}\text{ بجای }x}{f(x)} = x^2 \cdot \underbrace{\left[x^2 + \frac{1}{2}\right]^2}_{16} + 1 = 16x^2 + 1$$

$$\to f'(x) = 32x$$

$$(f \circ g(x))' = g'(x) \cdot f' \circ g(x) \quad \left\{\begin{array}{l} g'(x) = \left((x^2-1)^{-\frac{1}{3}}\right)' = -\frac{1}{3}(2x)(x^2-1)^{-\frac{4}{3}} \\ g'\left(\frac{3}{\sqrt{8}}\right) = -\frac{1}{3} \cdot \frac{3}{\sqrt{2}} \cdot \underbrace{\left(\frac{1}{8}\right)^{-\frac{4}{3}}}_{16} = -8\sqrt{2} \end{array}\right.$$

$$x = \frac{3}{\sqrt{8}} \to -8\sqrt{2} \cdot 32 \times 2 = \underset{\times 4}{-128\sqrt{2}}$$

$$\Rightarrow \frac{(f \circ g)'\left(\frac{3}{\sqrt{8}}\right)}{-128\sqrt{2}} = \underline{\underline{+4}}$$

۸) $$f(x) = f(x \pm 5) \to g(x) = f(x+1) + \underbrace{f(3x+10)}_{f(3x+5)} \to g'(x) = f'(x+1) + 3f'(3x+5)$$

$$x = -2 \to x + 1 = -1$$

$$\hookrightarrow 3x = -6 \to 3x + 5 = -1 \to g'(-2) = f'(-1) + 3f'(-1) = \underline{\underline{4}}$$

۹) $$\lim_{h\to 0} \frac{(f(5-h)-1)}{h} \cdot \frac{f(5-h)-2}{5-h} = \frac{f(5)-2}{5} \cdot (-f'(5)) = -\frac{1}{5} \cdot \frac{-25}{12} = \underline{\underline{\frac{5}{12}}}$$

$$f(x) = (x-5)\sqrt[3]{x+3} + \sqrt[3]{x+3} \longrightarrow f'(5) = \left(\sqrt[3]{x+3} + \frac{1}{3}(x+3)^{-\frac{2}{3}}\right)\Big|_{x=5} = 2 + \frac{1}{12} = \frac{25}{12}$$

$10)\quad y=(x-2)^2+1 \rightarrow y'=2x-4$

$4y-2x=1 \rightarrow y=\frac{1}{2}x+\frac{1}{4} \rightarrow m=\frac{1}{2}$ ، عمود $\rightarrow m'=-2=2x-4 \rightarrow x_0=1$

$y_0=(1-2)^2+1 \rightarrow y_0=2 \rightarrow \underline{A\left|\begin{matrix}1\\2\end{matrix}\right.}$