۱-

14,5

$$f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \xrightarrow{\text{پیوستگی در } R} \lim_{x\to1^+} = f(1) = \lim_{x\to1^-} \to \boxed{a+1=b} \bigstar$$

مشتق ناپذیر در R به جز یک نقطه → تابع $x<1$ : $a+\sqrt{x^2} = a+|x|$ → نقطه‌ی مشتق ناپذیری نقطه‌ی $x=0$ است . → در $x=1$ مشتق پذیر

۲

$$\to f'_+(1)=f'_-(1) \to x = b\times\frac{2}{3}\times x^{-\frac{1}{3}} \to \frac{2b}{3}=1 \to b=\frac{3}{2} \xrightarrow{\bigstar} a=\frac{1}{2}$$

$a+b=2$

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۲-

$$f(x)=\frac{|2x-3|}{\sqrt[3]{x^2}} \to f(1)=\frac{3-2x}{\sqrt[3]{x^2}} \to f'(1)=\frac{-2\sqrt[3]{x^2}-\frac{2}{3}x^{-\frac{1}{3}}\times(3-2x)}{(\sqrt[3]{x^2})^2}=-\frac{10}{3}$$

$$g(x)=\frac{\cancel{\sqrt{x^2-4x+4}}\to|x-2| + \sqrt{3x}}{\sqrt[3]{x^2}} \to g(1)=\frac{2-x+\sqrt{3x}}{\sqrt[3]{x^2}} \to g'(1)=\frac{\sqrt[3]{x^2}\left(-1+\frac{3}{2\sqrt{3x}}\right)-(2-x+\sqrt{3x})\left(\frac{2}{3\sqrt[3]{x}}\right)}{(\sqrt[3]{x^2})^2}$$

۲

$$\to g'(1)=-1+\frac{3}{2\sqrt{3}}-(1+\sqrt{3})\left(\frac{2}{3}\right) = -1-\frac{2}{3}-\frac{2}{\sqrt{3}}+\frac{3}{2\sqrt{3}} = -\frac{5}{3}-\frac{1}{2\sqrt{3}}$$

$$=\frac{-10-\sqrt{3}}{6} \qquad f'(1)-2g'(1) = -\frac{10}{3}-\left(\frac{-10-\sqrt{3}}{3}\right)=\frac{\sqrt{3}}{3}$$

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۳-

$$f(x)=\begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq a\end{cases} \xrightarrow{\text{در } R \text{ پیوسته}} \lim_{x\to a^+} f(x) = f(a) = \lim_{x\to a^-} f(x) \to \boxed{ab+c=\frac{1}{a}} \bigstar$$

۲

در R مشتق پذیر

$$f'(x)=\begin{cases} b & x<a \\ -\frac{1}{x^2} & x\geq a\end{cases} \to \boxed{b=-\frac{1}{a^2}} \xrightarrow{\bigstar} -\frac{1}{a}+c=\frac{1}{a} \to c=\frac{2}{a} \to ac=2$$

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۴-

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نقطه‌ی گوشه‌ای → $f_+(a) = f_-(a)$

$m=0 \to$ مشتق همه‌جا صفر است ✗

$m=1 \to f'_-(a)=0 \;,\; f'_+(a)=1$ ✓ $\to \boxed{m=1}$

$\geq 2 \to f'_-(a)=0 \;,\; f'_+(a)=m\underbrace{(x-a)}_{a-a=0}{}^{m-1}$ ✗

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۵-

1,5

$$(f\circ g)'(a)=g'(a)\cdot f'(g(a)) \to g'(-\sqrt[3]{2})\,f'(g(-\sqrt[3]{2})) = (f\circ g)'(-\sqrt[3]{2})$$

$$g(x)=\frac{1}{x^3-|x^3|} \to g(-\sqrt[3]{2})=\frac{1}{-2-2}=-\frac{1}{4}$$

کامل ایی

$$f(x)=\frac{1}{\sqrt[3]{x-|x|}} \to f\left(-\frac{1}{4}\right)=\frac{1}{\sqrt[3]{-\frac{1}{8}}}=\frac{1}{-\frac{1}{2}}=-2$$

-۶

$x=3$ در $y=-ax-5$ مماس $f(x)$ است $\rightarrow \begin{cases} f(3)=-3a-5 \\ f'(3)=-a \end{cases} \longrightarrow f'(3)-f(3)=2 \rightarrow 2a+5=2 \rightarrow 2a=-3$

$a=-\frac{3}{2}$

$f(3)=-3a-5=\frac{9}{2}-5=-\frac{1}{2}$ , $f'(3)=\frac{3}{2}$ $\Big\}$ $\frac{f(3)}{f'(3)}=\frac{-\frac{1}{2}}{\frac{3}{2}}=-\frac{1}{3}$

۴

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-۷

$(f\circ g)'(x)=g'(x).f'(g(x)) \xrightarrow{x=\frac{3}{\sqrt{8}}} =\frac{64\times 4}{\sqrt{8}\times 3\sqrt[3]{\frac{49}{4}}}$

۱٫۲۵

$g(x)=\frac{1}{\sqrt[3]{x^2-1}} \rightarrow g'(x)=\frac{2x}{3\sqrt[3]{(x^2-1)^2}} \rightarrow g'\left(\frac{3}{\sqrt{8}}\right)=\frac{\frac{6}{\sqrt{8}}}{3\sqrt[3]{\left(\frac{28}{8}\right)^2}}$

$g\left(\frac{3}{\sqrt{8}}\right)=\frac{1}{\sqrt[3]{\frac{1}{8}}}=2 \rightarrow f'(2):\ f(x)=\Big(x\underbrace{\left[x^2+\frac{1}{2}\right]}_{4.5}\Big)^2+1 \xrightarrow{x=2} f(x)=16x^2+1$

$f'(x)=32x$

$f'(2)=64$

$\frac{(f\circ g)'\left(\frac{3}{\sqrt{8}}\right)}{-128\sqrt{2}}=\frac{\cancel{64}\times\cancel{4}}{\cancel{-128}\sqrt{2}\times\underbrace{\sqrt{8}}_{\sqrt{16}=4}\times 3\times\sqrt[3]{\frac{49}{4}}}=\frac{-1}{4\sqrt[3]{\frac{49}{4}}}$

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-۸

$g(x)=f(x+1)+f(3x+10) \rightarrow g(-2)=f(-1)+f(4) \xrightarrow[T=5]{f\ \text{متناوب}} f(-1)=f(4) \rightarrow f'(-1)=f'(4)$

$\rightarrow g'(-2)=2f'(-1)=3$

۱٫۷۵

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-۹

$\lim_{h\to 0}\frac{f^2(5-h)-3f(5-h)+2}{h(5-h)} \xleftrightarrow{f(5-h)=a} a^2-3a+2=(a-2)(a-1) \rightarrow \lim_{h\to 0}\underbrace{\frac{f(5-h)-2}{h}}_{f'(5)}\times\lim_{h\to 0}\frac{\overset{f(5)}{f(5-h)}-1}{5-h}$

$h\to 0 \Rightarrow f(5-h)=f(5)=\sqrt[3]{8}=2$

۴

$=\boxed{\frac{1}{5}\times f'(5)} \rightarrow f(x)=(x-3)\sqrt[3]{x+3} \rightarrow f'(x)=\sqrt[3]{x+3}+\frac{(x-3)}{3\sqrt[3]{(x+3)^2}} \rightarrow f'(5)=2+\frac{1}{12}=\frac{25}{12}$

$\hookrightarrow \frac{1}{5}f'(5)=\frac{5}{12}$

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-۱۰

$4y-3x-1 \rightarrow y=\frac{3x}{4}+1 \xrightarrow[\text{عرض}]{x=\frac{1}{2}}$ (۲)

$f(x)=x^2-4x+5 \rightarrow f'(x)=2x-4 \rightarrow f'(x)=-2 \rightarrow 2x-4=-2 \rightarrow x=1$

۴

$x=1 \rightarrow f(1)=2 \rightarrow$ نقطه $(1,2)$

کامل

عالی!

5

$$g(-\sqrt[3]{2}) = \frac{1}{(-\sqrt[3]{2})^3 - |-\sqrt[3]{2}|^3} = \frac{1}{-2-2} = -\frac{1}{4}$$

$$f(-\tfrac{1}{4}) = \frac{1}{\sqrt[3]{-\frac{1}{4} - \frac{1}{4}}} = (-\tfrac{1}{2})^{-\frac{1}{3}} = -(2)^{-1 \times -\frac{1}{3}} = -2^{\frac{1}{3}} = -\sqrt[3]{2}$$

$\longrightarrow f \circ g(x) = x$

$$g'(x) \times f'(g(x)) = (x)' = 1$$

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7

$$(f \circ g)' = g'(\tfrac{3}{\sqrt{8}}) \times f'(g(\tfrac{3}{\sqrt{8}}))$$

$$g(\tfrac{3}{\sqrt{8}}) = \frac{1}{\sqrt[3]{\frac{9}{8} - 1}} = 2 \longrightarrow g'(\tfrac{3}{\sqrt{8}}) \times f'(2)$$

$$g'(\tfrac{3}{\sqrt{8}}) = -\frac{1}{3}(x^2-1)^{-\frac{4}{3}} \times 2x = \frac{6}{\sqrt{8}} \times -\frac{1}{3} \times (\tfrac{9}{8} - 1)^{-\frac{4}{3}} = -\frac{2}{\sqrt{8}} \times (2^{-3})^{-\frac{4}{3}}$$

$$= -\frac{2^5 \times \sqrt{2}}{4} = -8\sqrt{8}$$

$$f(x),\ x = 2 \to f(x) = x[\varepsilon]^2 + 1 = 3x^2 + 1 \to f'(2) = 32(2) = 64$$

$$\frac{-8\sqrt{2} \times \cancel{64}}{-\underset{2}{\cancel{128}}\sqrt{2}} = 4$$

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8

$$g'(x) = f'(x+1) + 3f'(3x+10)$$

$$\xrightarrow{x=-2} g'(-2) = f'(-1) + 3f'(4) \xrightarrow{f'(4) = f'(-1)} g'(-2) = 4f'(-1) = 4 \times \frac{3}{2} = \boxed{6}$$