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$$f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \xrightarrow{\text{پیوستگی در } \mathbb{R}} \lim_{x\to1^+} = f(1) = \lim_{x\to1^-} \to \boxed{a+1=b} \star$$

مشتق تابع در $\mathbb{R}$ به جز یک نقطه $\to$ تابع در $x<1$: $a+\sqrt{x^2} = a+|x| \to$ نقطه‌ی مشتق‌ناپذیری $\to$ در $x=1$ مشتق‌پذیر

نقطه‌ی $x=0$ است.

$$\to f'_+(1)=f'_-(1) \to x = b\times\frac{2}{3}\times x^{-\frac13} \to \frac{2b}{3}=1 \to b=\frac32 \to a=\frac12$$

$$a+b=2$$

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$$f(x)=\frac{|2x-4|}{\sqrt[3]{x^2}} \to f(1)=\frac{4-2x}{\sqrt[3]{x^2}} \to f'(1)=\frac{-2\sqrt[3]{x^2}-\frac23 x^{-\frac13}\times(4-2x)}{(\sqrt[3]{x^2})^2} = -\frac{10}{3}$$

$$g(x)=\frac{\cancel{\sqrt{x^2-4x+4}}^{\,|x-2|}+\sqrt{3x}}{\sqrt[3]{x^2}} \to g(1)=\frac{2-x+\sqrt{3x}}{\sqrt[3]{x^2}} \to g'(1)=\frac{\sqrt[3]{x^2}\left(-1+\frac{3}{2\sqrt{3x}}\right)-(2-x+\sqrt{3x})\left(\frac{2}{3\sqrt[3]{x}}\right)}{(\sqrt[3]{x^2})^2}$$

$$\to g'(1) = -1+\frac{3}{2\sqrt3}-(1+\sqrt3)\left(\frac23\right) = -1-\frac23-\frac{2}{\sqrt3}+\frac{3}{2\sqrt3} = -\frac53-\frac{1}{2\sqrt3}$$

$$= \frac{-10-\sqrt3}{6} \qquad f'(1)-2g'(1) = -\frac{10}{3}-\left(\frac{-10-\sqrt3}{3}\right) = \frac{\sqrt3}{3}$$

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$$f(x)=\begin{cases} bx+c & x<a\\ \frac1x & x\geq a\end{cases} \xrightarrow{\text{پیوسته در } \mathbb{R}} \lim_{x\to a^+} f(x) = f(a) = \lim_{x\to a^-} f(x) \to \boxed{ab+c=\frac1a} \star$$

در $\mathbb{R}$ مشتق‌پذیر

$$f'(x)=\begin{cases} b & x<a \\ -\frac{1}{x^2} & x>a\end{cases} \to \boxed{b=-\frac{1}{a^2}} \xrightarrow{\star} -\frac1a + c = \frac1a \to c=\frac2a \to ac=2$$

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$$(f\circ g)'(a) = g'(a).f'(g(a)) \to g'(-\sqrt[3]{2})\, f'(g(-\sqrt[3]{2})) = (f\circ g)'(-\sqrt[3]{2})$$

$$g(x)=\frac{1}{x^3-|x^3|} \to g(-\sqrt[3]{2}) = \frac{1}{-2-2} = -\frac14$$

$$f(x)=\frac{1}{\sqrt[3]{x-|x|}} \to f\left(-\frac14\right)=\frac{1}{\sqrt[3]{-\frac18}}=\frac{1}{-\frac12}=-2$$

ناقص است!

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در $x=3$ ، $y=-ax-5$ بر $f(x)$ مماس است $\rightarrow \begin{cases} f(3)=-3a-5 \\ f'(3)=-a \end{cases} \longrightarrow f'(3)-f(3)=2 \rightarrow 2a+5=2 \rightarrow 2a=-3$

$a=-\frac{3}{2}$

$f(3)=-3a-5=\frac{9}{2}-5=-\frac{1}{2}$

$f'(3)=\frac{3}{2}$

$\frac{f(3)}{f'(3)}=\frac{-\frac{1}{2}}{\frac{3}{2}}=-\frac{1}{3}$

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$(f\circ g)'(x)=g'(x).f'(g(x)) \xrightarrow{x=\frac{3}{\sqrt8}} =\frac{64\times 6}{\sqrt8\times 3\sqrt[3]{\frac{49}{4}}}$

$g(x)=\frac{1}{\sqrt[3]{x^2-1}} \rightarrow g'(x)=\frac{2x}{3\sqrt[3]{(x^2-1)^2}} \rightarrow g'\left(\frac{3}{\sqrt8}\right)=\frac{\frac{6}{\sqrt8}}{3\sqrt[3]{\left(\frac{1}{8}\right)^2}}$

$g\left(\frac{3}{\sqrt8}\right)=\frac{1}{\sqrt[3]{\frac{1}{8}}}=2 \rightarrow f'(2):\ f(x)=\left(x\underbrace{\left[x^2+\frac{1}{2}\right]}_{4,5}\right)^2+1 \xrightarrow{x=2} f(x)=16x^2+1$

$f'(x)=32x$

$f'(2)=64$

$\frac{(f\circ g)'\left(\frac{3}{\sqrt8}\right)}{-128\sqrt2}=\frac{\cancel{64\times6}}{\cancel{-128}\sqrt2\times\sqrt8\times3\sqrt[3]{\frac{49}{4}}}=\frac{-1}{4\sqrt[3]{\frac{49}{4}}}$

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$g(x)=f(x-1)+f(3x+10) \rightarrow g(-2)=f(-1)+f(4) \xrightarrow[T=5]{f\ \text{متناوب}} f(-1)=f(4) \rightarrow f'(-1)=f'(4)$

$\rightarrow g'(-2)=2f'(-1)=3$

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$\lim\limits_{h\to0}\frac{f^2(5-h)-3f(5-h)+2}{h(5-h)} \xleftrightarrow{f(5-h)=a} a^2-3a+2=(a-2)(a-1) \rightarrow \lim\limits_{h\to0}\underbrace{\frac{f(5-h)-2}{h}}_{f'(5)}\times\lim\limits_{h\to0}\frac{f(5-h)-1}{5-h}$

$h\to0 \Rightarrow f(5-h)=f(5)=\sqrt[3]{8}=2$

$=\boxed{\frac{1}{5}\times f'(5)} \rightarrow f(x)=(x-3)\sqrt[3]{x+3} \rightarrow f'(x)=\sqrt[3]{x+3}+\frac{(x-3)}{3\sqrt[3]{(x+3)^2}} \rightarrow f'(5)=2+\frac{1}{12}=\frac{25}{12}$

$\frac{1}{5}f'(5)=\frac{5}{12}$

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$4y-3x-1 \rightarrow y=\frac{3x}{4}+1 \xrightarrow[\text{شیب}]{x=\frac{1}{2}} (2)$

$f(x)=x^2-4x+5 \rightarrow f'(x)=2x-4 \rightarrow f'(x)=-2 \rightarrow 2x-4=-2 \rightarrow x=1$

$x=1 \rightarrow f(1)=2 \rightarrow$ نقطه $(1,2)$

طاها