

$f(x) = \begin{cases} a + \sqrt{x^2} & ; x < 1 \\ b\sqrt{x^2} & ; x \geq 1 \end{cases}$
 $\xrightarrow{x=1} a + 1 = b \rightarrow a - b = -1$

نقطه‌ای ۰ مشتق پذیر نیست $\rightarrow$ نقطه‌ای ۱ مشتق پذیر باشد $\rightarrow \frac{1}{|x|} = \frac{2b}{3\sqrt{x}} \rightarrow 3 = 2b$
 $b = 1.5$
 $a = 0.5$
 $a + b = 2$ ✓

$f(x) = \frac{|2x-4|}{\sqrt{x^2}} \xrightarrow{x=1} \frac{4-2x}{\sqrt{x^2}} \rightarrow f'(x) = \frac{-2\sqrt{x^2} - \frac{2(4-2x)}{3\sqrt{x}}}{\sqrt{x^2}}$

$g(x) = \frac{\sqrt{x^2-4x+4} + \sqrt{3x}}{\sqrt{x^2}} \xrightarrow{x=1} \frac{2-x+\sqrt{3x}}{\sqrt{x^2}} \rightarrow g'(x) = \frac{(-1+\frac{3}{2\sqrt{x}})(\sqrt{x^2}) - \frac{2(2-x+\sqrt{3x})}{3\sqrt{x}}}{\sqrt{x^2}}$

$f'(1) - 2g'(1) = -\frac{10}{3} - 2(-\frac{10-\sqrt{3}}{3}) = -\frac{10}{3} + \frac{10+\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$ ✓

$f(x) = \begin{cases} bx+c & ; x < a \\ \frac{1}{x} & ; x \geq a \end{cases}$

$\rightarrow b \cdot a + c = \frac{1}{a} \rightarrow -\frac{1}{a} + c = \frac{1}{a}$
 $c = \frac{2}{a}$
 $b = \frac{-1}{a^2}$ (مشتق)
 $a < \frac{2}{a} \Rightarrow \frac{2}{a} = 2$ ✓

$f(x) = \begin{cases} b & ; x < a \\ b + (x-a)^m & ; x \geq a \end{cases}$

(a, b) نقطه‌ای گویباری $\rightarrow m \neq 0$
 $(x-a)^m \rightarrow m = 1$
 نقطه‌ای ۰ مشتق پذیر نیست $\rightarrow$ مشتق پذیر باشد $\rightarrow$ $m = 1$
 یک مقدار ✓

$g(-\sqrt[3]{2}) f'(g(-\sqrt[3]{2})) = (f \circ g(-\sqrt[3]{2}))' = 1$

$g(x) = \frac{1}{x^3 - |x^3|} \xrightarrow{x = -\sqrt[3]{2}} \frac{1}{2x^3} \xrightarrow{x = -\sqrt[3]{2}} f \circ g = \frac{1}{\sqrt[3]{\frac{1}{2x^3}}} = \frac{1}{\frac{1}{x}} = x$ ✓

$f(x) = \frac{1}{\sqrt[3]{x-|x|}} \xrightarrow{x = -\sqrt[3]{2}} \frac{1}{\sqrt[3]{2x}}$ ✓

$$y = -ax - a \rightarrow f \text{ بر } x \text{ مماس در نقطه ای به طول } 3 \rightarrow f(3) = -3a - a = -4a$$

$$f'(3) = -a$$

$$\frac{f(3)}{f'(3)} = \frac{-4a}{-a} = 4$$

$$f'(3) - f(3) = 7$$

$$-a - (-4a - a) = 7 \rightarrow -a + 4a + a = 7 \rightarrow 4a = 7 \rightarrow a = \frac{7}{4}$$

$$f(x) = (x[x^2 + \frac{1}{x}])^2 + 1 \xrightarrow{x = g(\frac{1}{\sqrt{x}})} (x[f, \omega])^2 + 1 = 19x^2 + 1$$

$$g(x) = \frac{1}{\sqrt{x^2 - 1}} \xrightarrow{x = \frac{1}{\sqrt{x}}} f \circ g = 19(x^2 - 1)^{-\frac{1}{2}} + 1 \rightarrow$$

$$\rightarrow (f \circ g)' = 19x^{-\frac{1}{2}} \cdot \frac{1}{\sqrt{x^2 - 1}} \cdot \frac{1}{2\sqrt{x}} = \frac{19}{2\sqrt{x} \sqrt{x^2 - 1}}$$

$$g(x) = f(x+1) + f(3x+10)$$

$$f'(-1) = \frac{3}{2} = f(4)$$

$$g'(x) = f'(x+1) + 3f'(3x+10)$$

$$g'(-2) = f'(-2+1) + 3f'(-2+10)$$

$$g'(-2) = f'(-1) + 3f'(4) \rightarrow g'(-2) = \frac{3}{2} + \frac{9}{2} = 6$$

$$f(x) = (x-4)\sqrt[3]{x+3} \rightarrow f(5) = 2 \xrightarrow{\text{در } x=5 \text{ مماس}}$$

$$f'(x) = \sqrt[3]{x+3} + (x-4) \frac{1}{3\sqrt[3]{(x+3)^2}} \rightarrow f'(5) = \frac{10}{3}$$

$$\lim_{h \rightarrow 0} \frac{f^2(a-h) - 2f(a-h) + 1}{h(a-h)} \xrightarrow{\text{HOP}} \lim_{h \rightarrow 0} \frac{-2f(a-h)f'(a-h) - 2f'(a-h)}{a-2h}$$

$$\rightarrow \frac{-2f(a)f'(a) - 2f'(a)}{a} = \frac{f'(a)}{a} = \frac{\frac{10}{3}}{5} = \frac{2}{3}$$

$$y = x^2 - 4x + 5 \xrightarrow{\text{شیب خط مماس}} 2x - 4 \rightarrow 2x - 4 = 2 \rightarrow x = 1$$

$$y - 3x = 1 \rightarrow \text{شیب} = \frac{1}{3} \rightarrow \text{شیب عمود} = -3$$

$$y = x^2 - 4x + 5 \xrightarrow{x=1} 1 - 4 + 5 = 2 \rightarrow (1, 2)$$

$$\lim_{h \rightarrow 0} \frac{(f(a-h) - 1)(f(a-h) - 1)}{h(a-h)} = -f'(a) \times \frac{(f(a) - 1)}{a}$$

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$$f'(a) = \sqrt[n+3]{n+3} + \frac{n-1}{\sqrt[n+3]{n+3}^2} \leadsto -f'(a) = -(\sqrt[n]{n} + \frac{1}{\sqrt[n]{n}^2}) = -\frac{2a}{12}$$

$$-\frac{2a}{12} \times \frac{2-1}{a} = \boxed{-\frac{a}{12}}$$

قانون مشتق $\rightarrow \frac{f(h) - f(a-h)}{h} = f'(a)$