① $f(x)=\begin{cases} a+\sqrt{x^2} & ; x<1 \\ b+\sqrt{x^2} & ; x\geq 1 \end{cases}$ ... $a+b=?$ (۲)

$\begin{cases} a+|x| & x<1 \\ b\sqrt{x^2} & x\geq 1 \end{cases}$ ... $\frac{1}{2x^2b}$ ... $\frac{2b}{2}=1 \rightarrow b=\frac{2}{2}$

$a=b-1 \rightarrow a=\frac{1}{2}$

② $f(x)=\frac{|x-1|+|x-3|}{\sqrt{x^2}}$ ، $g(x)=\frac{\sqrt{x^2-4x+4}+\sqrt{x^2}}{\sqrt{x^2}}$ ... $f'(1)g(2)-(1)g'(1)$

$x^2-4x+4=(x-2)^2$

$f'(1)-g(2)g'(1)=\left(\frac{-2\sqrt{x^2}}{x^2\sqrt{x^2}}\right)'$

$-\left(\frac{6}{2\sqrt{x^2}} \times \frac{x^2-2\sqrt{x^2}(3x)}{9x^4}\right) \rightarrow \frac{2\sqrt{x^2}}{6}=-\frac{2\sqrt{x^2}}{6}=-\frac{\sqrt{2}}{2}$ (۱)

③ $f(x)=\begin{cases} bx+c & ; x<a \\ \frac{1}{x} & ; x\geq a \end{cases}$ ... $x=0$ ... $-\frac{1}{x}+c=\frac{1}{x}$

$f'(x)=\begin{cases} b & x<a \\ -\frac{1}{x^2} & x>a \end{cases} \rightarrow b=-\frac{1}{x^2}$

$c=\frac{2}{x}=\frac{2}{a}$

$bx+c=\frac{1}{x}$

$x \rightarrow a$

$ac=2$ (۴)

④ $f(x)=\begin{cases} b & ; x<a \\ b+(x-a)^m & ; x\geq a \end{cases}$ ... $m(x-a)^{m-1}\neq 0$ ... $m=1$ (۳)

$f'(x)=\begin{cases} 0 & x<a \\ m(x-a)^{m-1} & x>a \end{cases}$ ... $a^+ \rightarrow m(0^+)^{m-1}\neq 0 \rightarrow m=1$

⑤ $f(x)=\frac{1}{\sqrt{x-|x|}}$ ، $g(x)=\frac{1}{x^2-|x|}$ ... $(gof)'(-2)=((-2)g)'(f(-2))$

$gof=\frac{1}{\sqrt{x-|x|-x^2+|x|}}=\frac{1}{\sqrt{-x^2}}=\frac{1}{\sqrt{4x^2}}=\frac{1}{2x}$

$\frac{1}{-4}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{4}$ (۱)

⑥ $f(x)-f(x)=2$ ... $y=-ax-b$ ... $\frac{f(x)}{f'(x)}=?$

$f(x)=-x^2-b$

$f'(x)=-a$

$-a+2a+b=2a+b=2 \rightarrow 2a=-2 \rightarrow a=-\frac{2}{2}$

$\rightarrow f(x)=+\frac{2}{4}-b=-\frac{1}{2}$

$\rightarrow f'(x)=\frac{2}{2}$ ... $\frac{-\frac{1}{2}}{\frac{2}{2}}=-\frac{1}{2}$ (۵)

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