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صالح عالی

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$T=5 \rightarrow f'(-1)=f'(4)$

$g'(x)=f'(x+1)+3f'(3x+10) \rightarrow g'(-2)=f'(-1)+3f'(4)=\cancel{4}f'(-1)=\boxed{6}$

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$\lim_{h\to 0}\frac{f(5-h)-1}{5-h} \times \lim_{h\to 0}\frac{f(5-h)-\overbrace{\frac{5}{2}}^{f(5)}}{h}=-\frac{1}{5}\times\lim_{h\to 0}\frac{f(5-h)-f(5)}{-h}=$

$-\frac{1}{5}f'(5)=-\frac{1}{5}\times\frac{25}{12}=-\frac{5}{12}$

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$f'(x)=\sqrt[3]{x+5}+\frac{x-4}{3\sqrt[3]{(x+3)^2}} \xrightarrow{x=5} \frac{25}{12}$

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$2y=3x+1 \rightarrow m=\frac{3}{2} \rightarrow m_{\text{عمود}}=-2 \rightarrow 2x-4=-2 \rightarrow \begin{cases} x=1 \\ y=2 \end{cases}$

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صحیح نمائی

تکلیف — ریاضی ۳ (دوازدهم تجربی B)

به نام خدا

① $f(x^+) = f(x^-)$ در $x=0$ مشتق پذیر

$f'(x^+) = f'(x^-)$ پیوسته است

$$f(x)\begin{cases} a+|x| , & x<1 \\ b\sqrt[3]{x^2} , & x\geq 1 \end{cases}$$

نقطه مرزی باید

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$a+1=b$ (I) , $x=\dfrac{b\,2}{3\sqrt[3]{x}} \xrightarrow{x=1} b=\dfrac{3}{2} \to a=\dfrac{1}{2} , a+b=\underline{\underline{2}}$

② $f(1)=0 \to \dfrac{-2x+2}{\sqrt[3]{x^2}} \to f'(x)=\dfrac{-2(\sqrt[3]{x^2})-\left(\frac{2}{3\sqrt[3]{x}}\times(2x+2)\right)}{(\sqrt[3]{x^2})^2} \to f'(1)=\dfrac{-10}{3}$

$g(x)=\dfrac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}} \xrightarrow{x=1} g(x)=\dfrac{-x+2+\sqrt{3x}}{\sqrt[3]{x^2}} \to g'(x)=\dfrac{\left(\frac{3}{2\sqrt{3x}}-1\right)(\sqrt[3]{x^2})-\left(\frac{2}{3\sqrt[3]{x}}\times(-x+2+\sqrt{3x})\right)}{(\sqrt[3]{x^2})^2}$

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$\to g'(1)=\dfrac{3-4\sqrt{3}}{9} \to f'(1)-2g'(1)=\dfrac{-13+6\sqrt{3}}{3}$

③ مقدار و مشتق تابع در نقطه مرزی برابر است: $b=-\dfrac{1}{a^2} , ab+c=\dfrac{1}{a} \to 1=\underbrace{a^2b}_{-1}+ac \to \boxed{ac=2}$

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④ $f'_-(a)=0$ $\quad f'_+\neq f'_-$

$f'_+(a)$

۱,۷۵

$\underset{x\geq a}{f'(x)}=m(x-a)^{m-1}\neq 0 \to m=1$ یا $m=0$

⑤ $g(x) \xrightarrow{x=\sqrt[3]{-2}} \dfrac{1}{2x^3} , f(x)=\dfrac{1}{\sqrt[3]{2x}} \to (f\circ g)(x)=\dfrac{1}{\sqrt[3]{\frac{2}{2x^3}}}=x \Rightarrow$

$=-\sqrt[3]{2}$

$(f\circ g)'(x)=\underline{\underline{1}}$

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⑥ $f'(3)=-a , f(3)=-3a-5 \Rightarrow -a+2a+5=2 \to a=-3 \to \dfrac{f(3)}{f'(3)}=\dfrac{4}{3}$

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⑦ $f\circ g=\Bigg(\underbrace{\dfrac{1}{\sqrt[3]{x^2-1}}\left[\dfrac{1}{\sqrt[3]{x^2-1}}\right]}_{A}\Bigg)^2+1 \to (f\circ g)'=2\times A'\times A \Rightarrow f\circ g\left(\dfrac{3}{\sqrt{8}}\right)=$

$2\times\left(\dfrac{2x}{3\sqrt[3]{(x^2-1)^4}}\right)\times\left(\dfrac{1}{\sqrt[3]{x^2-1}}\right)^{\frac{1}{2}}=\dfrac{2}{\sqrt{8}\sqrt[3]{\frac{1}{64}}}=\dfrac{8}{\sqrt{8}}$

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$\to \dfrac{\sqrt{8}}{-128\sqrt{2}}=-\dfrac{1}{64}$

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$$f(x)=\frac{4-2x}{\sqrt[3]{x^2}} \to f'(1)=\frac{-2(\sqrt[3]{1})-\frac{2}{3}(1)^{-\frac{1}{3}}(4-2)}{\sqrt[3]{1}}=-\frac{10}{3}$$

$$g(x)=\frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}} \to g'(1)=\frac{\left(-1+\frac{3}{2\sqrt{3}}\right)(1)-\frac{2}{3\sqrt[3]{1}}(1+\sqrt{3})}{\sqrt[3]{1}}$$

$$g'(1)=-1+\frac{3\sqrt{3}}{6}-\frac{2}{3}-\frac{2\sqrt{3}}{3}=-\frac{5}{3}-\frac{\sqrt{3}}{6}$$

$$f'(1)-2g'(1)=-\frac{10}{3}+\frac{10}{3}+\frac{2\sqrt{3}}{6}=\boxed{\frac{\sqrt{3}}{3}}$$

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نقطه گوشه‌ای $\to f'_+(a)=f'_-(a)$

$m=0 \to$ مشتق هر دو طرف برابر ۰ است ✗

$m=1 \to f'_-(a)=0 \quad , \quad f'_+(a)=1$ ✓ $\to \boxed{m=1}$

$m\geq 2 \to f'_-(a)=0 \quad , \quad f'_+(a)=m(\underbrace{x-a}_{a-a=0})^{m-1}$ ✗

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$f(3)=-3a-5 \qquad f'(3)=-a \qquad -3\left(-\frac{3}{2}\right)-5=-\frac{1}{2}$

$-a-(-3a-5)=2 \to 2a=3 \to a=-\frac{3}{2} \to \frac{f(3)}{f'(3)}=\frac{-\frac{1}{2}}{-\frac{3}{2}}=\frac{1}{3}$

$$(f\circ g)' = g'\left(\frac{3}{\sqrt{8}}\right) \times f'\left(g\left(\frac{3}{\sqrt{8}}\right)\right)$$

$$g\left(\frac{3}{\sqrt{8}}\right) = \frac{1}{\sqrt[3]{\frac{9}{8}-1}} = 2 \longrightarrow g'\left(\frac{3}{\sqrt{8}}\right) \times f'(2)$$

$$g'\left(\frac{3}{\sqrt{8}}\right) = -\frac{1}{3}(x^2-1)^{-\frac{4}{3}} \times 2x = \frac{6}{\sqrt{8}} \times -\frac{1}{3} \times \left(\frac{9}{8}-1\right)^{-\frac{4}{3}} = -\frac{2}{\sqrt{8}} \times (2^{-3})^{-\frac{4}{3}}$$

$$= -\frac{2^5 \times \sqrt{2}}{4} = -8\sqrt{8}$$

$$f(x),\ x=2 \longrightarrow f(x) = x[4]^2+1 = 16x^2+1 \longrightarrow f'(2) = 32(2) = 64$$

$$\frac{-8\sqrt{2} \times 6\cancel{4}}{-1\cancel{2}8\sqrt{2}} = \boxed{4}$$

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