۵/۳/۵ ستاری - دوازدهم پیش B

بنام خدا

تکلیف شماره ۲۲

۱۹ آفرین

۱)

$f(x) = \begin{cases} a+\sqrt[]{x} & x<1 \\ b+\sqrt[3]{x^2} & x\geq 1 \end{cases}$

$f(x) = \begin{cases} a+|x| & x<1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases}$ $\quad 2 + 1 = b \;\; a - b = -1$

$b = \frac{3}{2} \;,\; a = \frac{1}{2}$

$f'(x) = \begin{cases} -1 & x\leq 0 \\ 1 & 0\leq x\leq 1 \\ \frac{2b}{3\sqrt[3]{x}} & x\geq 1 \end{cases}$ $\quad f' = \frac{2b}{3}$

$f(x) = \begin{cases} a-x & x<0 \\ a+x & 0\leq x\leq 1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases}$ $\quad b = \frac{3}{2} \rightarrow a+b = 2$

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۲)

$h(x) = \frac{|2x-4|}{\sqrt[3]{x^2}} = \frac{4-2x}{\sqrt[3]{x^2}} \rightarrow f'(x) = -2\sqrt[3]{x^2} \cdot (4x) \frac{2x}{3\sqrt[3]{x^4}}$

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$x=1 \rightarrow f'(1) = -\frac{1}{3} \;,\; g(x) = \frac{(2-x)+\sqrt[3]{x}}{\sqrt[3]{x^2}} \Rightarrow g'(x) = \frac{\left(-1+\frac{1}{3\sqrt[3]{x^2}}\right)\left(\sqrt[3]{x^2}\right) - \left(2-x+\sqrt[3]{x}\right)\left(\frac{2x}{3\sqrt[3]{x^4}}\right)}{\left(\sqrt[3]{x^2}\right)^2}$

$= \frac{-1 \ldots \sqrt{3}}{9}$

۳)

$f(x) = \begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq a \end{cases}$ $\quad x=a \rightarrow ab+c = \frac{1}{a}$

$\rightarrow -\frac{1}{x^2} = b \rightarrow b = -\frac{1}{a^2}$ $\quad -\frac{1}{a}+c = \frac{2}{a} = c$

$ac = 2$

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۴)

$f(x) = \begin{cases} b & x<a \\ b+(x-a)^m & m\geq 0 \end{cases}$ $\quad f'(x) = \begin{cases} 0 & x<0 \\ m(x-a)^{(m-1)} & x\geq 0 \end{cases}$

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$m-1=0 \rightarrow m=1$

۵)

$g'(-\sqrt{x}) \; f'(g(-\sqrt{2})) \cdot (fog(-\sqrt{2})) \rightarrow x = -\sqrt{2} \rightarrow f(x) = \frac{1}{\sqrt[3]{2x}} \rightarrow fog(x) = \frac{1}{\sqrt[3]{\frac{2}{2x^2}}}$

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$(fog)' = (x)' = 1 \rightarrow x' = 1$

مرحله سازی

-۶

$f(x) = -3x - 5 ,\ f'(x) = -x \rightarrow f'(x) - f(x) = -x + 3x + 5 = 2$

$x = -\frac{3}{2} \rightarrow f(x) = -\frac{1}{2} ,\ f'(x) = \frac{3}{2} \rightarrow \frac{f(x)}{f'(x)} = -\frac{\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$

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-۷

$f(x) = \left(x\left[x^2 + \frac{1}{2}\right]\right)^2 + 1 \quad f(g(x)) \rightarrow \left(\frac{1}{\sqrt[3]{x^2-1}}\left[4 + \frac{1}{2}\right]\right)^2 + 1$

$g(x) = \frac{1}{\sqrt[3]{x^2-1}}$

$\left(\frac{6}{\sqrt[3]{x^2-1}}\right)^2 + 1 \rightarrow \frac{8 \times 2x}{3\left(\sqrt[3]{x^2-1}\right)^3} \quad \left(\frac{6}{\sqrt[3]{x^2-1}}\right.$

$\frac{9 \times \frac{3}{\sqrt[3]{8}} \times \frac{6}{4}}{-3 \times \frac{1}{4}} = 96\sqrt{8} \quad \frac{-96\sqrt{8}}{128\sqrt{8}} = -\frac{1}{2}$

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-۸

$g(x) = f(x+1) + f(3x+10) \quad g'(x) = f(x+1)' + 3f(3x+10)' \rightarrow g'(-2) = f'(1) + 3f'(4)$

$g'(-2) = \frac{3}{2} + \frac{9}{2} = 6$

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-۹

$\lim \frac{(f(5-x))^2 - 3f(5-x) + 2}{5x - x^2} \xrightarrow{\text{هوپیتال}} \frac{2(-f(5-x))f(5-x) + 3f(5-x)}{5 - 2x}$

$\frac{f'(5)}{5} = \frac{-\frac{25}{12}}{5} = -\frac{5}{12}$

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-۱۰

$6y = 3x + 1 \rightarrow y = \frac{1}{2}x + \frac{1}{6} \rightarrow m = \frac{1}{2} \rightarrow m' = -2 \rightarrow (x^2 - 4x + 5)' = -2$

$2x - 4 = -2 \rightarrow x = 1 \rightarrow x = 1 \rightarrow x^2 - 4x + 5 = 2 \quad \left|\begin{matrix}1\\2\end{matrix}\right.$

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۶

$$f(x) = \frac{۴-۲x}{\sqrt[۳]{x^۲}} \rightarrow f'(۱) = \frac{-۲(\sqrt[۳]{۱}) - \frac{۲}{۳}(۱)^{-\frac{۱}{۳}}(۴-۲)}{\sqrt[۳]{۱}} = -\frac{۱۰}{۳}$$

$$g(x) = \frac{|x-۲| + \sqrt{۳x}}{\sqrt[۳]{x^۲}} \rightarrow g'(۱) = \frac{\left(-۱ + \frac{۳}{۲\sqrt{۳}}\right)(۱) - \frac{۲}{۳\sqrt[۳]{۱}}(۱+\sqrt{۳})}{\sqrt[۳]{۱^۲}}$$

$$g'(۱) = -۱ + \frac{۳\sqrt{۳}}{۶} - \frac{۲}{۳} - \frac{۲\sqrt{۳}}{۳} = -\frac{۵}{۳} - \frac{\sqrt{۳}}{۶}$$

$$f'(۱)\ ۳g'(۱) = -\frac{۱۵}{۳} + \frac{۱۰}{۳} + \frac{۳\sqrt{۳}}{۶} = \boxed{\frac{\sqrt{۳}}{۲}}$$

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$$(fog)' = g'\left(\frac{۳}{\sqrt{۸}}\right) \times f'\left(g\left(\frac{۳}{\sqrt{۸}}\right)\right)$$

$$g\left(\frac{۳}{\sqrt{۸}}\right) = \frac{۱}{\sqrt[۳]{\frac{۹}{۸}-۱}} = ۲ \rightarrow g'\left(\frac{۳}{\sqrt{۸}}\right) \times f'(۲)$$

$$g'\left(\frac{۳}{\sqrt{۸}}\right) = -\frac{۱}{۳}(x^۲-۱)^{-\frac{۴}{۳}} \times ۲x = \frac{۶}{\sqrt{۸}} \times -\frac{۱}{۳} \times \left(\frac{۹}{۸}-۱\right)^{-\frac{۴}{۳}} = -\frac{۲}{\sqrt{۸}} \times (۲^{-۳})^{-\frac{۴}{۳}}$$

$$= -\frac{۲^۵ \times \sqrt{۲}}{۴} = -۸\sqrt{۸}$$

$$f(x),\ x = ۲ \rightarrow f(x) = x[٤]^۲ + ۱ = ۱۶x^۲ + ۱ \rightarrow f'(۲) = ۳۲(۲) = ۶۴$$

$$\frac{-۸\sqrt{۲} \times \cancel{۶۴}}{-\underset{۲}{\cancel{۱۲۸}}\sqrt{۲}} = \boxed{۴}$$