۱۹٫۲۵ آفرین

نام و نام خانوادگی ...امیرعلی تنوایی‌راد... پاسخنامه تشریحی تکلیف شماره ...۲۲... کلاس ...دوازدهم تجربی B...

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$$f(x)=\begin{cases} a+|x| \\ b\sqrt[3]{x^2}\end{cases}$$ در نقطه صفر مشتق‌پذیر نیست

$$f'(x)=\begin{cases} \frac{2x}{2\sqrt{x^2}} & x<1 \\ \frac{2}{3}b\times x^{-\frac{1}{3}} & x\geq 1\end{cases}$$

در نقطه ۱ مشتق‌پذیر است $\Longrightarrow a+1=b$

$1=\frac{2}{3}b\times 1 \to b=\frac{3}{2} \quad a=\frac{1}{2}$

$a+b=2$

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$$g(x)=\frac{2-x+\sqrt{3x}}{\sqrt[3]{x^2}} \qquad f(x)=\frac{4-2x}{\sqrt[3]{x^2}} \qquad (f-2g)(x)=\frac{4-2x}{\sqrt[3]{x^2}}-\frac{4-2x+2\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$=\frac{-2\sqrt{3x}}{\sqrt[3]{x^2}} \xrightarrow{\text{مشتق}} \frac{\frac{-6}{2\sqrt{3x}}\times\sqrt[3]{x^2}+\frac{2}{3}x^{-\frac{1}{3}}\times 2\sqrt{3x}}{\sqrt[3]{x^4}} \xrightarrow{x=1} \frac{-6}{2\sqrt{3}}\times 1+\frac{2}{3}\times 2\sqrt{3}$$

$$\frac{-3}{\sqrt{3}}+\frac{4\sqrt{3}}{3}=\frac{-3\sqrt{3}}{3}+\frac{4\sqrt{3}}{3}=\frac{\sqrt{3}}{3}$$

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$$f'(x)=\begin{cases} b & x<a \\ \frac{-1}{x^2} & x>a\end{cases} \to f'_-(a)=f'_+(a) \to b=\frac{-1}{a^2} \to a^2b=-1$$

$$f(a^+)=f(a^-) \Rightarrow ab+c=\frac{1}{a} \xrightarrow{\times a} a^2b+ac=1 \to -1+ca=1 \to ac=2$$

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$f'_-(a)=0$

$f'_+(a)\neq 0$

$$f'(x)=\begin{cases} 0 & x<a \\ m(x-a)^{m-1} & x>a\end{cases}$$

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پس تنها یک مقدار حاصل می‌شود!

$m(x-a)^{m-1}\neq 0$

$m=1 \to f'_+(a)=m=1$

$m=0 \to m\times\frac{1}{x-a} \xrightarrow{x=a^+} m\times\frac{1}{0^+}=+\infty \to f'_+(a)=\infty$

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$$g(x)=\frac{1}{2x^3} \qquad f(x)=\frac{1}{\sqrt[3]{2x}} \qquad g'(x)=-1\times\frac{1}{(2x^3)^2}\times 6x^2$$

$$f'(x)=-\frac{2}{3}(2x)^{-\frac{4}{3}} \qquad g'(-\sqrt[3]{2})=-1\times\frac{1}{16}\times 6\times\sqrt[3]{4}$$

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$$f'\left(\frac{-1}{4}\right)=-\frac{2}{3}\times\left(\frac{-1}{2}\right)^{-\frac{4}{3}}=-\frac{2}{3}\times\frac{1}{\left(\frac{-1}{2}\right)^{\frac{4}{3}}}=-\frac{2}{3}\times\frac{1}{\sqrt[3]{\frac{1}{16}}}$$

$$-1\times\frac{1}{16}\times 6\times\sqrt[3]{4}\times-\frac{2}{3}\times\frac{1}{\sqrt[3]{\frac{1}{16}}}=-1$$

$f(3) = -3a - 5 \quad f'(3) = -a \quad f'(3) - f(3) = -a - (-3a - 5) = 2 \longrightarrow$

$-a + 3a + 5 = 2 \rightarrow 2a = -3 \rightarrow a = \frac{-3}{2}$ ۶

$f(3) = -3 \times \frac{-3}{2} - 5 = \frac{9}{2} - \frac{10}{2} = \frac{-1}{2}$

$f'(3) = -a = \frac{3}{2} \longrightarrow \frac{f(3)}{f'(3)} = \frac{\frac{-1}{2}}{\frac{3}{2}} = \frac{-1}{3}$ ۲

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$f(g(x)) = g'(x) \times f'(g(x)) \longrightarrow g'\left(\frac{3}{\sqrt{8}}\right) \times f'(2)$ ۷۵

$g'(x) = \frac{-1}{3} \times \frac{1}{\sqrt[3]{(x^2-1)^4}} \times 2x \Rightarrow g'\left(\frac{3}{\sqrt{8}}\right) = \frac{-1}{\cancel{3}} \times \frac{1}{16_{8}} \times \frac{6}{\sqrt{8}} = \frac{-1}{\sqrt{8}}$ ۷

$f(x) = (x \times 4)^2 + 1 = 16x^2 + 1 \longrightarrow f'(x) = 8(4x) \longrightarrow f'(2) = 64$

$fog'\left(\frac{3}{8}\right) = 64 \times \frac{-1}{\sqrt{8}} = \frac{-64}{\sqrt{8}} = \frac{-64}{2\sqrt{2}} = \frac{-32}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-32\sqrt{2}}{2} = -16\sqrt{2} \rightarrow \frac{-16\sqrt{2}}{-128\sqrt{2}} = \frac{1}{8}$

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$g'(x) = f'(x+1) + 3 \times f'(3x+10) \longrightarrow g'(-2) = f'(-1) + 3 \times f'(4) \qquad f'(-1) = f'(-1+5)$

$g'(-2) = \frac{3}{2} + 3 \times \frac{3}{2} = \frac{3}{2} + \frac{9}{2} = 6$ ۲

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$\xrightarrow{\text{Hop}} \frac{-2f(5-h) \times f'(5-h) + 3f'(5-h)}{-2h+5} = \frac{-2f(5) \times f'(5) + 3f'(5)}{5} = \frac{-4 \times \frac{25}{12} + \frac{25}{4}}{5} =$

$h \to 0 \qquad \frac{\frac{-25}{3} + \frac{25}{4}}{5} = \frac{\frac{-100+75}{12}}{5} = \frac{\frac{-25}{12}}{5} = \frac{-25}{60} = \boxed{\frac{-5}{12}}$ ۹

$f'(x) = 1 \times \sqrt[3]{x+3} + \frac{1}{3} \times \frac{1}{\sqrt[3]{(x+3)^2}}(x-4) \longrightarrow f'(5) = 2 + \frac{1}{3} \times \frac{1}{4} \times 1 = 2 + \frac{1}{12} = \frac{25}{12}$

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$6y = 3x + 1 \longrightarrow y = \frac{1}{2}x + \frac{1}{6} \longrightarrow m_{\text{مماس}} = -2$

$y' = 2x - 4 = -2 \rightarrow 2x = 2 \rightarrow x = 1 \quad \frac{1}{2}$ ۲ ۱۰

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$$g(-\sqrt[۳]{۲}) = \frac{1}{(-\sqrt[۳]{۲})^{۳} - |-\sqrt[۳]{۲}|^{۳}} = \frac{1}{-۲-۲} = \frac{-1}{۴}$$

$$f(-\tfrac{1}{۴}) = \frac{1}{\sqrt[۳]{-\frac{1}{۴} - \frac{1}{۴}}} = (-\tfrac{1}{۲})^{-\frac{1}{۳}} = -(۲)^{-1 \times -\frac{1}{۳}} = -۲^{\frac{1}{۳}} = -\sqrt[۳]{۲} \rightarrow fog(x) = x$$

$$g'(x) \times f'(g(x)) = (x)' = 1$$

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$$(fog)' = g'(\tfrac{۳}{\sqrt{۸}}) \times f'(g(\tfrac{۳}{\sqrt{۸}}))$$

$$g(\tfrac{۳}{\sqrt{۸}}) = \frac{1}{\sqrt[۳]{\frac{۹}{۸} - 1}} = ۲ \rightarrow g'(\tfrac{۳}{\sqrt{۸}}) \times f'(۲)$$

$$g'(\tfrac{۳}{\sqrt{۸}}) = -\tfrac{1}{۳}(x^{۲}-1)^{-\frac{۴}{۳}} \times ۲x = \frac{۶}{\sqrt{۸}} \times -\frac{1}{۳} \times (\tfrac{۹}{۸} - 1)^{-\frac{۴}{۳}} = -\frac{۲}{\sqrt{۸}} \times (۲^{-۳})^{-\frac{۴}{۳}}$$

$$= -\frac{۲^{۵} \times \sqrt{۲}}{۴} = -۸\sqrt{۸}$$

$$f(x),\ x = ۲ \rightarrow f(x) = x[\varepsilon]^{۲} + 1 = ۳x^{۲} + 1 \rightarrow f'(۲) = ۳۲(۲) = ۹۶$$

$$\frac{-۸\sqrt{۲} \times ۹۶}{-۱۹۲\sqrt{۲}} = \boxed{۴}$$