۱۹/۵ آفرین

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۱) $f'(3) = \text{شیب خط مماس} = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \boxed{\frac{4}{3}}$ ✓

۲) $m = \frac{1}{3} \longrightarrow$ معادله خط مماس $= y = \frac{1}{3}x + \frac{4}{3}$

$\Rightarrow \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \longrightarrow 3\sqrt{ax-1} = x + 4$

$\Rightarrow 9(ax-1) = x^2 + 8x + 16 \Rightarrow x^2 + (8-9a)x + 25 = 0$ ✓

$\Delta = 0 \Rightarrow (8-9a)^2 - 4(25) = 0 \Rightarrow 8 - 9a = \pm 10$

$\Rightarrow \begin{cases} a = 2 \checkmark \longrightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \boxed{3} \\ a = -\frac{2}{9} \longrightarrow \text{غیرقابل قبول} \end{cases}$

۳) $\text{شیب} = \frac{3}{4} \Rightarrow y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2}$ ✓

$\xrightarrow{x=1} \frac{9+3m}{16} = \frac{3}{4} \longrightarrow m = 2 \text{ و } n = 1 \longrightarrow \boxed{m+n = 3}$

(۴)

$$f(x)=\frac{(۳-\sin x)(۹+\sin^۲ x+۳\sin x)}{(۳-\sin x)(۳+\sin x)} \Rightarrow f(x)=\frac{۹+\sin^۲ x+۳\sin x}{۳+\sin x}$$

۱,۷۵

$$g(x)=\frac{۳}{۳+\sin x} \rightarrow g'(x)=\frac{-\cos x}{(۳+\sin x)^۲}$$

$$f'(x)=\frac{(۲\sin x\cos x+۳\cos x)(۳+\sin x)-\cos x(۹+\sin^۲ x+۳\sin x)}{(۳+\sin x)^۲}$$

$$g'\left(\frac{۵\pi}{۳}\right)=\frac{-\frac{۱}{۲}}{\left(۳+\frac{\sqrt{۳}}{۲}\right)^۲}$$

$\frac{۳۳}{۴}$

$$f'\left(\frac{۵\pi}{۳}\right)=\frac{\left(-\frac{\sqrt{۳}}{۲}+\frac{۳}{۲}\right)\left(۳-\frac{\sqrt{۳}}{۲}\right)-\frac{۱}{۲}\left(۹+\frac{۳}{۴}-\frac{۳\sqrt{۳}}{۲}\right)}{\left(۳+\frac{\sqrt{۳}}{۲}\right)^۲}$$

(۵)

$$f(x)\xrightarrow{x>0}-\frac{1}{\sqrt[۵]{۲x}}$$

$$g(x)\xrightarrow{x>0}\frac{1}{۲x^۵}$$

$$fog(x)=-x \Rightarrow fog'(x)=-1$$

$$\Rightarrow fog'(\sqrt[۵]{۳})=-1$$

۲

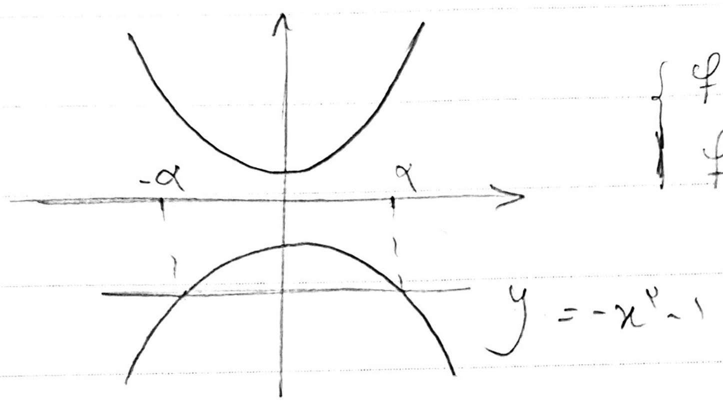
(6)

$$g(x)=\frac{f(x)-1}{x} \Rightarrow \frac{1+\sin^2 x-2\sin x}{1+\sin^2 x+2\sin x}-1=\frac{-4\sin x}{1+\sin^2 x+2\sin x}$$

$$\Rightarrow \lim_{x\to 0}\frac{\frac{-4\sin x}{1+\sin^2 x+2\sin x}}{x} \xrightarrow{\text{[illegible]}} \lim \frac{\frac{-4x}{1+x^2+2x}}{x}$$

$$\lim_{x\to 0}\frac{-4}{1+x^2+2x} \Rightarrow \boxed{-4}$$

(7)



$$\begin{cases} f'(\alpha)\cdot f'(-\alpha)=-1 \\ f(x)=-x^2-1 \Rightarrow f'(x)=-2x \end{cases}$$

$$\Rightarrow (-2\alpha)(+2\alpha)=-1$$

$$\Rightarrow \boxed{\alpha^2=\frac{1}{4}}$$

$$\Rightarrow f(\alpha)=f(-\alpha)=-\alpha^2-1=-\frac{1}{4}-1=\boxed{-\frac{5}{4}}$$

فاصله $=\frac{5}{4}=\boxed{1{,}25}$

(۸) خطا $\rightarrow ax-b$ از مبدأ می‌گذرد $\rightarrow b=0$ ، $f(x)$ بر نمودار مماس است $\rightarrow f(x)=ax$, $f'(x)=a$

$$2\sqrt{x}\,(4x^2+3)=ax \rightarrow \left(\frac{1}{\sqrt{x}}\right)(4x^2+3)+(2\sqrt{x})(8x)=a=\frac{20x^2+3}{\sqrt{x}}$$

$$(2\sqrt{x})(4x^2+3)=x\times\frac{20x^2+3}{\sqrt{x}} \longrightarrow 8x^2+6=20x^2+3$$

$$\longrightarrow 12x^2-3=0 \Rightarrow x=+\frac{1}{2}\ \checkmark \Rightarrow \boxed{a=8\sqrt{2}}$$

$-\frac{1}{2}\times$

(۲)

(۹) $ax+b$ از مبدأ مختصات می‌گذرد $\rightarrow b=0$ ، $f(x)$ بر نمودار مماس است $\rightarrow f(x)=ax$, $f'(x)=a$

$$\frac{\sqrt{x}}{-2x^2+x+1}=ax \quad , \quad a=\frac{4x^2-x+1}{(2\sqrt{x})(-2x^2+x+1)}$$

در نقطه $A=f\left(\frac{1}{2}\right)=\frac{\sqrt{2}}{2}$

(۲)

$$\frac{\sqrt{x}}{-2x^2+x+1}=\left(\frac{x}{2\sqrt{x}}\right)\left(\frac{4x^2-x+1}{(-2x^2+x+1)^2}\right) \rightarrow 4x^2-x+1=-4x^2+2x+2$$

$$\boxed{x=\frac{1}{2}}$$

(۱۰)

$$f(x)=(x[x])^3 \ , \ g(x)=\frac{1}{\sqrt{x^2-1}} \rightarrow fog(x)=$$

(۱/۷۵)

$$\left(\left(\frac{1}{\sqrt{x^2-1}}\right)\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^3$$

مشتق → $(fog)'(x)=g'(x)\cdot f'(g(x))=\boxed{-\frac{\sqrt{5}}{200}}$

$$۳g - f(x) = \frac{۹}{۳+\sin x} - \frac{(\cancel{۳-\sin x})(۹+\sin^{۲} x+۳\sin x)}{(\cancel{۳-\sin x})(۳+\sin x)} = \frac{-\sin x(\cancel{\sin x+۳})}{\cancel{\sin x+۳}}$$ ۴

$$\to -\sin x \xrightarrow{\text{مشتق}} (۳g-f)'(x) = -\cos x \rightsquigarrow -\cos\left(\frac{۵\pi}{۳}\right) = \boxed{-\frac{۱}{۲}}$$

$$g(x) = (x^{۲}-۱)^{-\frac{۱}{۲}} \longrightarrow g'(x) = -\frac{۱}{۲}(۲x)(x^{۲}-۱)^{-\frac{۳}{۲}}$$ ۱۰

$$g'\left(\frac{\sqrt{۵}}{۲}\right) = -\frac{۱}{۲}(\sqrt{۵})\left(\frac{۵}{۴}-۱\right)^{-\frac{۳}{۲}} \longrightarrow -\frac{\sqrt{۵}}{۲}\left(\underset{۸}{\cancel{۲^{-۲(-\frac{۳}{۲})}}}\right) = -۴\sqrt{۵}$$

$$g\left(\frac{\sqrt{۵}}{۲}^{-}\right) = \frac{۱}{\sqrt{\frac{۵^{-}}{۴}-۱}} = \frac{۱}{\sqrt{\frac{۱}{۴}^{-}}} = \frac{۱}{\frac{۱}{۲}^{-}} = ۲^{+}$$

$$f'(۲^{+}) = ((۲x)^{۳})' = ۲۴x^{۲} = ۲۴ \times ۴$$

$$fog'\left(\frac{\sqrt{۵}}{۲}^{-}\right) = -۴\sqrt{۵} \times ۲۴ \times ۴ \xrightarrow{\div -۴۸\sqrt{۵}} \frac{\overset{۲}{\cancel{۲۴}} \times \cancel{۴} \times -۴\sqrt{۵}}{-\cancel{۴۸}\sqrt{۵}} = ۸$$