امین جلالی

(۱) $f'(۳) = \text{شیب خط} = \frac{\Delta y}{\Delta x} = \frac{۵-۱}{۳-۰} = \boxed{\frac{۴}{۳}}$

(۲) $m = \frac{۱}{۳} \rightarrow \text{معادله خط} \Rightarrow y = \frac{۱}{۳}x + \frac{۴}{۳}$

$$\Rightarrow \sqrt{ax-۱} = \frac{۱}{۳}x + \frac{۴}{۳} \rightarrow ۳\sqrt{ax-۱} = x+۴$$

$$\Rightarrow ۹(ax-۱) = x^۲ + ۸x + ۱۶ \Rightarrow x^۲ + (۸-۹a)x + ۲۵ = ۰$$

$$\Delta = ۰ \Rightarrow (۸-۹a)^۲ - ۴(۲۵) = ۰ \Rightarrow ۸-۹a = \pm ۱۰$$

$$\Rightarrow \begin{cases} a = ۲ \checkmark \rightarrow f(x) = \sqrt{۲x-۱} \Rightarrow f(۵) = \boxed{۳} \\ a = -\frac{۲}{۹} \rightarrow \text{غیرقابل قبول} \end{cases}$$

(۳) $\text{شیب} = \frac{۳}{۴} \Rightarrow y' = \frac{(۲x+m)(x+۲) - (x^۲+mx+۱)}{(x+۲)^۲}$

$$x=۱ \rightarrow \frac{۹+۳m}{۱۶} = \frac{۳}{۴} \rightarrow m = ۲ \text{ و } n = ۱ \rightarrow \boxed{m+n = ۳}$$

$$f(x)=\frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} \Rightarrow f(x)=\frac{9+\sin^2 x+3\sin x}{3+\sin x}$$ (۱۶)

$$g(x)=\frac{3}{3+\sin x} \longrightarrow g'(x)=\frac{-\cos x}{(3+\sin x)^2}$$

$$f'(x)=\frac{(2\sin x\cos x+3\cos x)(3+\sin x)-\cos x(9+\sin^2 x+3\sin x)}{(3+\sin x)^2}$$

$$g'\left(\frac{9\pi}{2}\right)=\frac{-\frac{1}{2}}{\left(3+\frac{\sqrt{3}}{2}\right)^2}$$

$$f'\left(\frac{\cos\pi}{3}\right)=\frac{\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\left(9+\frac{3}{\frac{15}{3\sqrt{3}}}-\frac{3\sqrt{3}}{2}\right)}{\left(3+\frac{\sqrt{3}}{2}\right)^2}$$

(۱۷)

$$f(x)\xrightarrow{x>0}-\frac{1}{\sqrt[3]{2x}}$$

$$g(x)\xrightarrow{x>0}\frac{1}{2x^3}$$

$$\left.\begin{array}{l} f_{og}(x)=-x \Rightarrow f_{og}(x)=-1 \\ \Rightarrow f_{og}'(\sqrt[3]{3})=-1 \end{array}\right\}$$

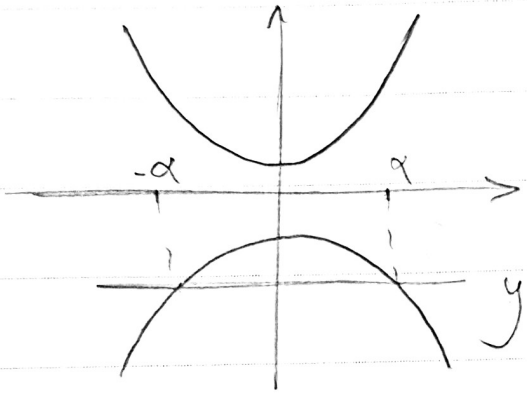
(4)

$$g(x) = \frac{f(x)-1}{x} \Rightarrow \frac{1+\sin^2 x - 4\sin x}{1+\alpha\sin^2 x + 4\beta\sin x} - 1 = \frac{-4\sin x}{1+\sin^2 x + 4\sin x}$$

$$\Rightarrow \lim_{x\to 0} \frac{\frac{-4\sin x}{1+\alpha\sin^2 x + 4\beta\sin x}}{x} \xrightarrow{(\text{Sil})} \lim \frac{\frac{-4x}{1+x^2+4x}}{x}$$

$$\lim_{x\to 0} \frac{-4}{1+x^2+4x} \Rightarrow \boxed{-4}$$

(V)



$$\begin{cases} f'(\alpha) - f'(-\alpha) = -1 \\ f(x) = -x^2 - 1 \Rightarrow f'(x) = -2x \end{cases}$$

$$y = -x^2 - 1 \Rightarrow (-2\alpha)(+2\alpha) = -1$$

$$\Rightarrow \alpha^2 = \frac{1}{4}$$

$$\Rightarrow f(\alpha) = f(-\alpha) = -\alpha^2 - 1 = -\frac{1}{4} - 1 = \boxed{\frac{-5}{4}}$$

$$\text{بر حسب } b = \frac{5}{4} = \boxed{1,25}$$

(۸) $f(x) = ax$, $f'(x) = a$ ← برنمودار $f(x)$ مماس است $b = 0$ ← از مبدأ میگذرد $ax - b$ ← خط

$$2\sqrt{x}\,(4x^2+3) = ax \rightarrow \left(\frac{1}{\sqrt{x}}\right)(4x^2+3) + (2\sqrt{x})(8x) = a = \frac{20x^2+3}{\sqrt{x}}$$

$$(2\sqrt{x})(4x^2+3) = x \times \frac{20x^2+3}{\sqrt{x}} \longrightarrow 8x^2+6 = 20x^2+3$$

$$\longrightarrow 12x^2 - 3 = 0 \Rightarrow x = +\frac{1}{2} ✓ \Rightarrow \boxed{a = 8\sqrt{2}}$$

یا $-\frac{1}{2}$ ×

(4) $f(x) = ax$, $f'(x) = a$ ← برنمودار $f(x)$ مماس است $b = 0$ ← از مبدا مختصات میگذرد $ax + b$

$$\frac{\sqrt{x}}{-2x^2+x+1} = ax \quad , \quad a = \frac{4x^2-x+1}{(2\sqrt{x})(-2x^2+x+1)} \qquad \text{عرض نقطه } A = f\left(\tfrac{1}{2}\right) = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{x}}{-2x^2+x+1} = \left(\frac{x}{2\sqrt{x}}\right)\left(\frac{4x^2-x+1}{(-2x^2+x+1)^2}\right) \rightarrow 4x^2-x+1 = -4x^2+2x+2$$

$$\boxed{x = \frac{1}{2}}$$

(۱۰)

$$f(x) = (x[x])^3 \ , \ g(x) = \frac{1}{\sqrt{x^2-1}} \rightarrow fog(x) =$$

$$\left(\left(\frac{1}{\sqrt{x^2-1}}\right)\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^3$$

مشتق چپ ← $(fog)'_-(x) = g'(x) \cdot f'(g(x)) = \boxed{-\frac{\sqrt{5}}{200}}$