آفرین ۱۹٫۵

رادمهر جعفری

۱) ۲ $y = mx+1 = m(x-۳)+۵ = mx + \underbrace{۵-۳m}_{1} \rightarrow m = \frac{۴}{۳} = f'(۳)$

۲) $y=ax+b \rightarrow \begin{cases} ۲a+b=۳ \\ -a+b=1 \end{cases} \quad b=\frac{۴}{۳} ,\ a=\frac{1}{۳} \rightarrow y = \frac{1}{۳}(x+۴)$

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$f(x) - \frac{1}{۳}(x+۴) = 0 \xrightarrow{\text{ریشه مضاعف}}$ مماس $\rightarrow ۳\sqrt{ax-1} = x+۴ \rightarrow ۹ax-۹ = x^۲+۸x+۱۶$

$\rightarrow x^۲ + (۸-۹a)x + ۲۵ = 0 \xrightarrow{\Delta=0} ۸-۹a=-۱۰ \quad , \quad ۸-۹a=+۱۰ \rightarrow a=-\frac{۲}{۹}$

$\rightarrow a=۲ : f(۵)=۳ \qquad f(۵) = \times$

۳) $f(x) = \frac{x^۲+۳x+(m-۳)x+۳(m-۳)-۳(m-۳)+1}{x+۳} = x+m-۳+\frac{۱۰-۳m}{x+۳}$

$\rightarrow$ ۲ $f'(x) = 1 - \frac{۱۰-۳m}{(x+۳)^۲} \rightarrow f'(1) = 1 - \frac{۱۰-۳m}{۱۶} = \frac{۳}{۴} = \frac{۱۲}{۱۶} \rightarrow ۱۶-(۱۰-۳m)=۱۲$

$= ۶+۳m \rightarrow m=۲$

$f(1) = \frac{1^۲+۲+1}{۴} = 1 = \frac{۳(1)+n}{۴} \rightarrow n=1 \rightarrow m+n=۳$

۴) $f(x) = \frac{(۳-\sin x)(۹+۳\sin x+\sin^۲ x)}{(۳-\sin x)(۳+\sin x)} = \frac{۹+۳\sin x+\sin^۲ x}{۳+\sin x}$

$h(x) = ۳g(x) - f(x) =$ ۲ $\frac{-۳\sin x - \sin^۲ x}{۳+\sin x} = -\sin x \rightarrow h'(x) = -\cos x \rightarrow h'(\frac{۵\pi}{۳}) = -\frac{1}{۲}$

۵) $x>0 \rightarrow g>0 \rightarrow fog \leq 0 : \quad x>0 ; g(x) = \frac{1}{۲}x^{-۵} , f(x) = -(۲x)^{-\frac{1}{۵}}$

$f \leq 0$

۲ $\rightarrow fog(x) = -(۲(\frac{1}{۲}x^{-۵}))^{-\frac{1}{۵}} = -x^{+1}$

$h(x) = -x \rightarrow h'(\sqrt{۳}) = -1$

۶) $g(x) = \frac{f(x)-1}{x-0} , f(0) = (\frac{-1}{1})^۲ = 1 \rightarrow \lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = f'(0)$

$h(x) =$ ۲ $\frac{۲x-1}{x+1} \rightarrow f(x) = h^۲(\sin x) \rightarrow f'(x) = ۲h'(\sin x)h(\sin x)\cos x$

$\rightarrow f'(0) = ۲h'(0)h(0) = ۲\times۲\times(-1) = -۴$

$h'(x) = \frac{۲}{(x+1)^۲} \rightarrow h'(0)=۲$

7) $y_1 = x^2+1 \xrightarrow{\text{قرینه}} y_2 = -x^2-1$

$y_2' = -2x$

$y = \alpha = -x^2-1 \to x = \pm\sqrt{-a-1}$ $\Bigg\}$ $+4x_1 \times x_2 = -1 = -(-a-1)\times 4$

$\to a = -\frac{5}{4} \to |-\frac{5}{4}| = \underline{1.25}$

راه حل کلی سوال ۸، ۹ : $f(x) = \sqrt{x}\times g(x),\ u = \sqrt{x},\ v = g(x)$

$u^2 = x \to 2uu' = 1$

$f(x) = uv = mx = mu^2 \xrightarrow{u\neq 0} mu = v$ (I)

$f'(x) = u'v + v'u = m \xrightarrow{\times u} mu = v'u^2 + uu'v = v'x + \frac{1}{2}v$ (II)

II,I $\Rightarrow v'x + \frac{1}{2}v = v \to v'x = \frac{1}{2}v \to \boxed{v = 2v'x}$

8) $v = 8x^2+6 \to 8x^2+6 = (16x)\times 2x = 32x^2 \to 24x^2 = 6$

$\to x^2 = \frac{1}{4} \xrightarrow{x>0} x = \frac{1}{2} \to f'(\frac{1}{2}) = m$

$f'(x) = vu' + uv'$ $\Big\{ v(\frac{1}{2}) = 8,\ v'(\frac{1}{2}) = 8,\ u(\frac{1}{2}) = \frac{\sqrt{2}}{2},\ u'(\frac{1}{2}) = \frac{\sqrt{2}}{2}$

$f'(\frac{1}{2}) = 8\times\frac{\sqrt{2}}{2} + 8\times\frac{\sqrt{2}}{2} = \underline{8\sqrt{2}}$

9) $v = \frac{1}{-2x^2+x+1} \to v' = \frac{4x-1}{(-2x^2+x+1)^2} \to v = 2v'x$

$\times(-2x^2+x+1)^2 \to -2x^2+x+1 = 2(4x-1)x = 8x^2-2x \to 10x^2-3x+1 = (5x+1)(2x-1)$

$x>0 \to x = \frac{1}{2}$ 2 $A_y = f(\frac{1}{2}) = \frac{\sqrt{\frac{1}{2}}}{1} = \frac{\sqrt{2}}{2}$

10) $x>1 \to g(x)$ نزولی است: اگر $\frac{\sqrt{10}}{3} < x < \frac{\sqrt{5}}{2} \to 3 > g(x) > 2 \to fog(x) = 8g^3(x)$

$(fog)'_-(x) = (8g^3(x))' = 8\times 3g'(x)\times g^2(x)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = -\frac{1}{2}\times 2x\times(x^2-1)^{-\frac{3}{2}}$

$g(\frac{\sqrt{5}}{2}) = 2,\ g'(\frac{\sqrt{5}}{2}) = -4\sqrt{5}$ 2

$\Bigg\}$ $(fog^3)'_-(\frac{\sqrt{5}}{2}) = 8\times -12\sqrt{5}\times 4 \xrightarrow{\div(-48\sqrt{5})} \underline{8}$

۲

$m = \frac{۲-۱}{۲+۱} = \frac{۱}{۳} \rightsquigarrow f'(x) = \frac{a}{۲\sqrt{ax-۱}} = \frac{۱}{۳} \rightsquigarrow ۳a = ۲\sqrt{ax-۱}$

مماس $= y = \frac{۱}{۳}x + \frac{۴}{۳} \rightsquigarrow x+۴ = ۳\sqrt{ax-۱} \rightsquigarrow x+۴ = \frac{۳a}{۲}(۳) = \frac{۹a}{۲}$

$x = ۴/۵a - ۴ \rightsquigarrow ۴/۵a - ۴ + ۴ = ۳\sqrt{a(۴/۵a-۴)-۱} \rightsquigarrow ۹a^۲ - ۱۶a - ۴ = ۰ \Rightarrow a = ۲ ✓$

یا $a = -\frac{۲}{۹}$ ✗

$f(a) = \sqrt{۱۰-۱} = \sqrt{۹} = \boxed{۳}$