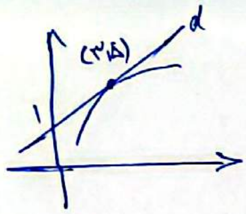
۱۷/۲۵

۱- $d:\ m=\frac{5-1}{3-0}=\frac{4}{3} \to f'(3)=\frac{4}{3}$



۲- $f(x)=\sqrt{ax-1} \to f'(x)=\frac{a}{2\sqrt{ax-1}}$

$(2,2),(-1,1) \to \frac{2-1}{2-(-1)}=\frac{1}{3} \to y=\frac{x}{3}+\frac{4}{3}$

$\begin{cases} f(A)=\frac{A}{3}+\frac{4}{3}=\sqrt{aA-1} \\ f'(A)=\frac{1}{3}=\frac{a}{2\sqrt{aA-1}} \quad * \end{cases}$

$\frac{A+4}{3}=\sqrt{Aa-1} \xrightarrow{*} \frac{a}{\frac{2(A+4)}{3}}=\frac{1}{3} \to 2A+8=9a \to A=\frac{9a-8}{2}$

$\xrightarrow{*} \frac{a}{2\sqrt{a\left(\frac{9a-8}{2}\right)-1}}=\frac{1}{3} \to \frac{a^2}{18a^2-16a-4}=\frac{1}{9} \to 9a^2-16a-4=0 \to a=\frac{16\pm\sqrt{256+144}}{18}$ ($\sqrt{256+144}=20$)

$\to a=\frac{36}{18}=2 \to f(x)=\sqrt{2x-1} \to f(5)=\sqrt{10-1}=\sqrt{9}=3$

۳- $y=\frac{x^2+mx+1}{x+3} \to y'=\frac{x^2+6x+3m-1}{(x+3)^2}=\frac{3}{4}$

$\xrightarrow{x=1} \frac{6+3m}{16}=\frac{3}{4} \to m=2 \to y=\frac{x^2+2x+1}{x+3} \to f(1)=\frac{4}{4}=1$

$\to$ نقطه $(1,1) \to 4=3+n \to n=1$

$m+n=3$

۴-

کاملا ا[illegible]

۵- $(f\circ g)'(x)=g'(x)\cdot f'(g(x)) \to g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3}))=(f\circ g)'(\sqrt[5]{3})$

۱/۵

$g'(\sqrt[5]{3})=\frac{1}{3+3}=\frac{1}{6}$

$f(x)=\frac{-1}{\sqrt[5]{2x}}=-2x^{-\frac{1}{5}} \to f'(x)=\frac{2}{5}x^{-\frac{6}{5}}$ $\Big\}$ $f'(\frac{1}{6})=\frac{2}{5}(6^{-1})^{-\frac{6}{5}}=\frac{2}{5}6^{\frac{6}{5}}=\frac{2}{5}\sqrt[5]{6^6}=\frac{12}{5}\sqrt[5]{6}$

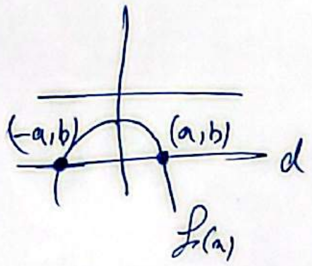
۶-

$$f(x) = \frac{\sin^2 x + 1 - 2\sin x}{\sin^2 x + 2\sin x + 1} = xg + 1 \to \frac{\cancel{\sin^2 x} + \cancel{1} - 2\sin x - \cancel{\sin^2 x} - \cancel{1} - 2\sin x}{\sin^2 x + 1 + 2\sin x} = xg \to xg = \frac{-4\sin x}{\sin^2 x + 2\sin x + 1}$$

$$\to g(x) = \frac{-4\sin x}{x(\sin^2 x + 2\sin x + 1)} \to \lim_{x\to 0} g(x) \underset{\sin x \sim x}{\xrightarrow{\text{هم ارزی}}} \lim_{x\to 0} \frac{-4x}{x^3 + 2x^2 + x} \underset{\text{Hop}}{\xrightarrow{\text{هم ارزی}}} \frac{-4}{3x^2 + 4x + 1} \quad -4$$

۷-

$$f(x) = x^2 + 1 \underset{\text{نسبت به محور } x}{\xrightarrow{\text{قرینه سازی}}} \boxed{f(x) = -x^2 - 1}$$



$$f(x) = -x^2 - 1 \to f'(x) = -2x$$

$$f'(a) = -2a$$
$$f'(-a) = 2a \xrightarrow{\text{عمود}} -2a = -\frac{1}{2a} \to 4a^2 = 1 \to a = \frac{1}{2} \to f(a) = f\left(\frac{1}{2}\right) = -\frac{1}{4} - 1 = -\frac{5}{4}$$

عرض از مبدا d

۸-

۹-

$$f(x) = \frac{\sqrt{x}}{-2x^2 + x + 1} \to f'(x) = \frac{\frac{\sqrt{x}}{2x}(-2x^2 + x + 1) + \sqrt{x}(4x - 1)}{(-2x^2 + x + 1)^2}$$

$$f(A) = aA \to \frac{\sqrt{A}}{-2A^2 + A + 1} = Aa \quad *$$

$$\boxed{\sqrt{A} = Aa(-2A^2 + A + 1)}$$

$$d:\ y = ax \to f(A) = aA$$

$$f'(A) = a \to \frac{\frac{\sqrt{A}}{2A}(-2A^2 + A + 1) + \sqrt{A}(4A - 1)}{(-2A^2 + A + 1)^2} = \frac{\frac{\sqrt{A}}{2A}(-2A^2 + A + 1 + 8A^2 - 2A)}{(-2A^2 + A + 1)^2}$$

۱۲A

$$\xrightarrow{*} \frac{(6A^2 - A + 1)\cancel{a}}{2(-2A^2 + A + 1)} = \cancel{a} \to 6A^2 - A + 1 = -4A^2 + 2A + 2 \to 10A^2 - 3A - 1 = 0 \to A = \frac{5}{10} = \frac{1}{2}$$

$$f(A) = f\left(\frac{1}{2}\right) = \frac{\sqrt{\frac{1}{2}}}{-1 + \frac{1}{2} + 1} = \frac{\sqrt{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{\frac{1}{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

۱۰-

$$f'(g(x)) = g'(x) \cdot f'(g(x))$$

$$g'(x) = -\frac{1}{2} \times 2x \times \frac{1}{(x^2 - 1)\sqrt{x^2 - 1}} \to g'\left(\frac{\sqrt{5}}{2}\right) = -4\sqrt{5}$$

$$f'(g(x)) = f'(2) = 24x^2$$

دقت! ن

$$= \frac{24 \times 4 \times -4\sqrt{5}}{-48\sqrt{5}} = -8$$

+

۵

$$g'(x) \times f'(g(x)) = (fog)'(x)$$

$$x>۰ \longrightarrow g(x) = \frac{۱}{۲x^۵} \longrightarrow f(x) = \frac{-۱}{\sqrt[۵]{۲x}} \rightsquigarrow fog(x) = \frac{-۱}{\sqrt[۵]{۲(\frac{۱}{۲x^۵})}}$$

$$fog(x) = -x \longrightarrow fog'(x) = \boxed{-۱} \rightsquigarrow fog'(\sqrt[۵]{۳}) = ۱$$

۸

$$f(x) = ۸x^{\frac{۵}{۲}} + ۹x^{\frac{۱}{۲}} \longrightarrow f'(x) = ۲۰ \cdot x^{\frac{۳}{۲}} + ۴.۵x^{-\frac{۱}{۲}}$$

معادله‌ی خط مماس در نقطه‌ی $x=a$ برابر است با:

$$y - ۲\sqrt{a}(۴a^۲+۳) = \frac{۲۰a^۲+۳}{\sqrt{a}}(x-a)$$

$$x,y=۰ \rightsquigarrow \cancel{-}۲\sqrt{a}(۴a^۲+۳) = \frac{۲۰a^۲+۳}{\sqrt{a}}(\cancel{-}a) \rightsquigarrow ۲\cancel{a}(۴a^۲+۳) = ۲۰a^۲+۳(\cancel{a})$$

$$۸a^۲+۶ = ۲۰a^۲+۳ \longrightarrow ۱۲a^۲ = ۳ \longrightarrow \boxed{a = \pm\frac{۱}{۲}} \rightsquigarrow a>۰ \rightsquigarrow a = \frac{۱}{۲}$$

$$m = ۲۰(۲^{-۱ \times \frac{۳}{۲}}) + ۳(۲^{-۱(\frac{-۱}{۲})}) = \boxed{۸\sqrt{۲}}$$

۱۰

$$g(x) = (x^۲-۱)^{-\frac{۱}{۲}} \longrightarrow g'(x) = -\frac{۱}{۲}(۲x)(x^۲-۱)^{-\frac{۳}{۲}}$$

$$g'(\frac{\sqrt{۵}}{۲}) = -\frac{۱}{۲}(\sqrt{۵})(\frac{۵}{۴}-۱)^{-\frac{۳}{۲}} \longrightarrow -\frac{\sqrt{۵}}{۲}(\underbrace{\cancel{۲^{-۲(-\frac{۳}{۲})}}}_{۸}) = -۴\sqrt{۵}$$

$$g(\frac{\sqrt{۵}}{۲}^-) = \frac{۱}{\sqrt{\frac{۵}{۴}^- - ۱}} = \frac{۱}{\sqrt{\frac{۱}{۴}^-}} = \frac{۱}{\frac{۱}{۲}^-} = ۲^+$$

$$f'(۲^+) = ((۲x)^۳)' = ۲۴x^۲ = ۲۴ \times ۴$$

$$fog'(\frac{\sqrt{۵}}{۲}^-) = -۴\sqrt{۵} \times ۲۴ \times ۴ \xrightarrow{\div -۴۸\sqrt{۵}} \frac{\cancel{۲۴}^{۲} \times \cancel{۴} \times -۴\sqrt{۵}}{-\cancel{۴۸}\sqrt{۵}} = ۸$$