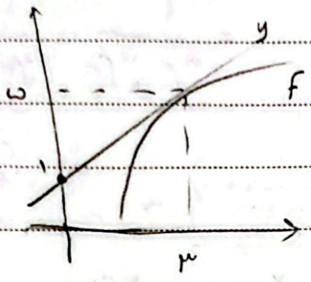
به نام خدا ۱۹ آذر

تکلیف شماره ۲٤ کیوان مکارم شب



(۱) $\begin{bmatrix}3\\5\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix} \to m=\frac{5-1}{3-0}=\frac{4}{3}$

$y=\frac{4}{3}x+1 \to y'(3)=f'(3)=\boxed{\frac{4}{3}}$ ✓

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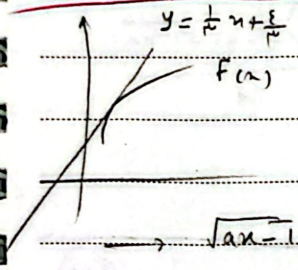

(۲) $\begin{bmatrix}2\\2\end{bmatrix},\begin{bmatrix}-1\\1\end{bmatrix} \to m=\frac{2-1}{2-(-1)}=\frac{1}{3} \to y=\frac{1}{3}x+\frac{4}{3}$

$\to$ ① $f(x)=g(x)$, ② $f'(x)=y'(x)$

$\to \sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3} \to \frac{3a}{2}=\frac{1}{3}x+\frac{4}{3} \to x=\frac{9a}{2}-4$ (I)

$\to \frac{a}{2\sqrt{ax-1}}=\frac{1}{3} \to \sqrt{ax-1}=\frac{3a}{2}$

(I) $\to \frac{a}{2\sqrt{a(\frac{9}{2}a-4)-1}}=\frac{1}{3} \to \frac{a^2}{4((\frac{9a^2}{2}-4a)-1)}=\frac{1}{9} \to 9a^2=18a^2-16a-4$

$\to 9a^2-16a-4=0 \to a=\frac{16\pm\sqrt{256+144}}{18} \Rightarrow a=\frac{36}{18}=2$ ✓

$a=\frac{-4}{18}=-\frac{2}{9}$ غ ق ق

چون $f(x)$ تعریف نمی‌شود

$\to f(x)=\sqrt{2x-1} \to f(5)=\sqrt{10-1}=\sqrt{9}=3$ ✓✓

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(۳) $y=\frac{3}{4}x+\frac{n}{4} \to y'=\frac{3}{4}=\frac{(2x+m)(x+3)-(1)(x^2+mx+1)}{(x+3)^2}$

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$\xrightarrow{x=1} \frac{(2+m)(4)-(m+2)}{16}=\frac{3}{4} \to 12=8+4m-m-2 \to 3m+6=12 \to \boxed{m=2}$

$\xrightarrow{x=1} y=f \to \frac{x^2+2x+1}{x+3}=\frac{3}{4}x+\frac{n}{4} \to \frac{1+2+1}{1+3}=\frac{3}{4}+\frac{n}{4} \to 4=3+n \to \boxed{n=1}$

$\to n+m=2+1=3$ ✓

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(٤) $3g(x) \Rightarrow 3\times\frac{3}{3+\sin x}=\frac{9}{3+\sin x}$

$\to 3g(x)-f(x)=\frac{9}{\sin x+3}-\frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)}=\frac{\cancel{9}-\cancel{9}-\sin^2 x-3\sin x}{\sin x+3}$

$=(3g-f)(x)=\frac{-\sin x(\cancel{\sin x+3})}{\cancel{3+\sin x}}=-\sin x \xrightarrow{\text{مشتق}} (3g-f)'(x)=-\cos x$

$\to (3g-f)'(\frac{5\pi}{3})=-\cos(\frac{5\pi}{3})=-\frac{1}{2}$ ✓

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۱۵ ۵

$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3})) = (fog)'(\sqrt[5]{3}) \rightarrow fog = \dfrac{-1}{\sqrt[5]{x^5+|x^5|+|x^5+|x^5||}}$

$\xrightarrow[x>0]{x=\sqrt[5]{3}} (fog)(x) = \dfrac{-1}{\sqrt[5]{4x^5}} = \dfrac{-1}{\sqrt[5]{4}\,x} \quad (fog)'(x) = \dfrac{1}{\sqrt[5]{4}\,x^2} = \dfrac{1}{\sqrt[5]{4}\,\sqrt[5]{9}} = \dfrac{1}{\sqrt[5]{36}}$

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$\lim\limits_{x\to 0} \dfrac{\left(\dfrac{-1+\sin x}{\sin x+1}\right)^2 - 1}{x} = \lim\limits_{x\to 0} \dfrac{\left(\dfrac{\sin x-1}{\sin x+1}-1\right)\left(\dfrac{\sin x-1}{\sin x+1}+1\right)}{x}$

$= \lim\limits_{x\to 0} \dfrac{\left(\dfrac{\cancel{\sin x}-1-\cancel{\sin x}-1}{\sin x+1}\right)\left(\dfrac{\sin x\cancel{-1}+\sin x\cancel{+1}}{\sin x+1}\right)}{x} = \lim\limits_{x\to 0} \dfrac{\left(\dfrac{-2}{\sin x+1}\right)\left(\dfrac{2\sin x}{\sin x+1}\right)}{x}$

هم ارزی $\lim\limits_{x\to 0} \dfrac{-4\cancel{x}}{(x+1)^2\cancel{(x)}} = \dfrac{-4}{(0+1)^2} = -4$ ✓ ۳

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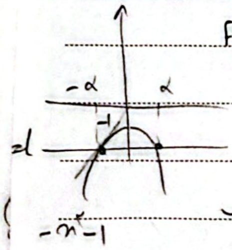

$f'(x) = -2x \rightarrow -2\alpha,\ 2\alpha \rightarrow -4\alpha^2 = -1 \rightarrow 4\alpha^2 = 1$

$\rightarrow \alpha^2 = \dfrac{1}{4} \rightarrow \alpha_1 = \dfrac{1}{2}$

$\alpha_2 = -\dfrac{1}{2} \rightarrow y = d = -(\pm\dfrac{1}{2})^2 - 1$

$\rightarrow y = d = -\dfrac{1}{4} - 1 = -\dfrac{5}{4} \xrightarrow[\text{تا مبدأ}]{\text{فاصله}} \dfrac{5}{4}$ ✓ ۲

۸ (۱۸)

$d: y = kx \xrightarrow{① \text{ مقدار برابر}} kx = 8\sqrt{x}\cdot x^2 + 6\sqrt{x}$

$f(x) = 2\sqrt{x}(4x^2+3) \xrightarrow[\text{مشتق برابر}]{②} k = \dfrac{4x^2+3}{\sqrt{x}} + 16\sqrt{x}\cdot x \ \xrightarrow{\times x}$

$kx = \dfrac{4x^3+3x}{\sqrt{x}} + 16\sqrt{x}\cdot x^2 \xrightarrow{①=②} 8\sqrt{x}\cdot x^2 + 6\sqrt{x} = \dfrac{4x^3+3x}{\sqrt{x}} + 16\sqrt{x}\cdot x^2 \ \times\sqrt{x}$

$8x^3 + 6x = 4x^3 + 3x + 16x^3 \rightarrow 12x^3 - 3x = 0 \rightarrow 3x(4x^2-1) = 0$

$\rightarrow x = 0$ ✗ $\quad x = \pm\dfrac{1}{2}$ ✓ $\xrightarrow{①} \dfrac{k}{2} = \dfrac{8\sqrt{2}}{2} \times \dfrac{1}{4} + \dfrac{6\sqrt{2}}{2} \rightarrow k = 8\sqrt{2}$ ✓ ۲ شیب

۱۹

$d: y = kx \quad \begin{cases} \xrightarrow[\text{مقدار برابر}]{①} kx = \dfrac{\sqrt{x}}{-2x^2+x+1} \\ \xrightarrow[\text{مشتق برابر}]{②} k = \dfrac{\dfrac{-2x^2+x+1}{2\sqrt{x}} + 4\sqrt{x}\cdot x - \sqrt{x}}{(-2x^2+x+1)^2} \end{cases}$

$f(x) = \dfrac{\sqrt{x}}{-2x^2+x+1}$

$\xrightarrow{\times x} kx = \dfrac{\dfrac{-2x^3+x^2+x}{2\sqrt{x}} + 4\sqrt{x}\cdot x^2 - \sqrt{x}\cdot x}{(-2x^2+x+1)^2}$

$\xrightarrow{①=②} \sqrt{x} = \dfrac{\dfrac{-2x^3+x^2+x}{2\sqrt{x}} + 4\sqrt{x}\cdot x^2 - \sqrt{x}\cdot x}{(-2x^2+x+1)} \xrightarrow{\times\sqrt{x}} -2x^3 + x^2 + x = \dfrac{-2x^3+x^2+x}{2} + \dfrac{4x^3 - x^2}{1}$

$\xrightarrow[\text{ساده سازی}]{\times 2} x(10x^2 - 3x - 1) = 0 \xrightarrow[x>0]{x\neq 0} x = \dfrac{3+\sqrt{40+9}}{20} = \dfrac{1}{2} \xrightarrow{①} \dfrac{k}{2} = \dfrac{\dfrac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} \rightarrow k = \sqrt{2}$

$\rightarrow y = \dfrac{1}{2}x \xrightarrow{x=1} \boxed{y = \dfrac{\sqrt{2}}{2}}$ ✓ ۲

Year .... Month .... Day .... Subject: ....

$fog(x) = \left(\frac{1}{\sqrt{x^2-1}} \left[\frac{1}{\sqrt{x^2-1}} \right] \right)^3 \quad \xrightarrow{x \to \frac{\sqrt{5}}{2}^-} \quad x < \frac{\sqrt{5}}{2} \to x^2 < \frac{5}{4}$ ۷۵ ۱۵

$\to x^2 - 1 < \frac{1}{4} \to \sqrt{x^2-1} < \frac{1}{2} \to \frac{1}{\sqrt{x^2-1}} > 2 \to \left[\frac{1}{\sqrt{x^2-1}} \right] = 2$

$\to (fog)(x) = \left(\frac{2}{\sqrt{x^2-1}} \right)^3 \to (fog)'(x) = 3 \left(\frac{2}{\sqrt{x^2-1}} \right)^2 \left(\frac{-2x}{\sqrt{x^2-1}} \right)$

$\xrightarrow{x = \frac{\sqrt{5}}{2}} 3 \left(\frac{2}{\frac{1}{2}} \right)^2 \left(\frac{-2\left(\frac{\sqrt{5}}{2}\right)}{\frac{1}{2}} \right) = 3(16)(-2\sqrt{5}) = -48\sqrt{5} \times 3$

$\to -48\sqrt{5}$ برابر ۳ ✓

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$g'(x) \times f'(g(x)) = (fog)'(x)$

$x>0 \longrightarrow g(x) = \frac{1}{۲x^{۵}} \longrightarrow f(x) = \frac{-1}{\sqrt[۵]{۲x}} \rightsquigarrow fog(x) = \frac{-1}{\sqrt[۵]{۲(\frac{1}{۲x^{۵}})}}$

$fog(x) = -x \longrightarrow fog'(x) = -1 \rightsquigarrow fog'(\sqrt[۵]{۳}) = 1$

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$g(x) = (x^{۲}-1)^{-\frac{1}{۲}} \longrightarrow g'(x) = -\frac{1}{۲}(۲x)(x^{۲}-1)^{-\frac{۳}{۲}}$

$g'(\frac{\sqrt{۵}}{۲}) = -\frac{1}{۲}(\sqrt{۵})(\frac{۵}{٤}-1)^{-\frac{۳}{۲}} \longrightarrow -\frac{\sqrt{۵}}{۲}(\underbrace{\cancel{۲^{-۲(-\frac{۳}{۲})}}}_{۸}) = -۴\sqrt{۵}$

$g(\frac{\sqrt{۵}}{۲}^{-}) = \frac{1}{\sqrt{\frac{۵}{٤}^{-}-1}} = \frac{1}{\sqrt{\frac{1}{٤}^{-}}} = \frac{1}{\frac{1}{۲}^{-}} = ۲^{+}$

$f'(۲^{+}) = ((۲x)^{۳})' = ۲۴x^{۲} = ۲۴ \times ٤$

$fog'(\frac{\sqrt{۵}^{-}}{۲}) = -۴\sqrt{۵} \times ۲۴ \times ٤ \xrightarrow{\div -۴۸\sqrt{۵}} \frac{\cancel{۲۴}^{۲} \times \cancel{۴} \times -۴\sqrt{۵}}{-\cancel{۴۸}\sqrt{۵}} = ۸$