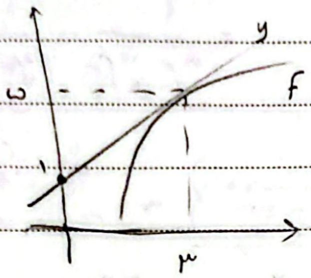
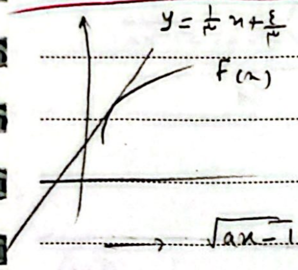
به نام خدا

تکلیف شماره ۲۴ — کیوان مکارم شب



$y \to \begin{bmatrix}3\\5\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \to m = \frac{5-1}{3-0} = \frac{4}{3}$ (۱)

$y = \frac{4}{3}x+1 \to y'(3) = f'(3) = \boxed{\frac{4}{3}}$ ✓



$\begin{bmatrix}2\\2\end{bmatrix},\begin{bmatrix}-1\\1\end{bmatrix} \to m = \frac{2-1}{2-(-1)} = \frac{1}{3} \to y=\frac{1}{3}x+\frac{4}{3}$ (۲)

$\to (1)\ f(x)=g(x),\ (2)\ f'(x)=y'(x)$

$\to \sqrt{ax-1} = \frac13 x+\frac43 \to \frac{3a}{2} = \frac13 x + \frac43 \to x = \frac{9a}{2} - 4$ (I)

$\to \frac{a}{2\sqrt{ax-1}} = \frac13 \to \sqrt{ax-1} = \frac{3a}{2}$

(I) $\to \frac{a}{2\sqrt{a(\frac92 a-4)-1}} = \frac13 \to \frac{a^2}{4((\frac{9a^2}{2}-4a)-1)} = \frac19 \to 9a^2 = 18a^2-16a-4$

$\to 9a^2-16a-4=0 \to a = \frac{16 \pm \sqrt{256+144}}{18} \to a=\frac{36}{18}=2$ ✓ ، $a = \frac{-4}{18} = -\frac29$ غ ق ق

$\to f(x)=\sqrt{2x-1} \to f(5)=\sqrt{10-1}=\sqrt9 = 3$ ✓✓ چون f(x) تعریف نمی‌شود.

$y=\frac34 x+\frac n4 \to y'=\frac34 = \frac{(2x+m)(x+3)-(1)(x^2+mx+1)}{(x+3)^2}$ (۳)

$\xrightarrow{x=1} \frac{(2+m)(4)-(m+2)}{16} = \frac34 \to 12 = 8+4m-m-2 \to 3m+6=12 \to \boxed{m=2}$

$\xrightarrow{x=1} y=f \to \frac{x^2+2x+1}{x+3} = \frac34 x+\frac n4 \to \frac{1+2+1}{1+3} = \frac34+\frac n4 \to 4 = 3+n \to \boxed{n=1}$

$\to n+m = 2+1 = 3$ ✓

$3g(x) \Rightarrow 3\times\frac{3}{3+\sin x} = \frac{9}{3+\sin x}$ (۴)

$\to 3g(x)-f(x) = \frac{9}{\sin x+3} - \frac{\cancel{(3-\sin x)}(9+\sin^2x+3\sin x)}{\cancel{(3-\sin x)}(3+\sin x)} = \frac{\cancel{9}-\cancel{9}-\sin^2 x-3\sin x}{\sin x+3}$

$=(3g-f)(x) = \frac{-\sin x\cancel{(\sin x+3)}}{\cancel{3+\sin x}} = -\sin x \xrightarrow{\text{مشتق}} (3g-f)'(x) = -\cos x$

$\to (3g-f)'(\frac{5\pi}{3}) = -\cos(\frac{5\pi}{3}) = -\frac12$ ✓

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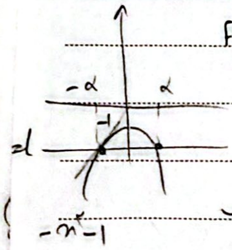
$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3})) = (f\circ g)'(\sqrt[5]{3}) \to f\circ g = \dfrac{-1}{\sqrt[5]{x^5+|x^5|+|x^5+|x^5||}}$ ۵

$x=\sqrt[5]{3}$, $x>0$: $(f\circ g)(x) = \dfrac{-1}{\sqrt[5]{4x^5}} = \dfrac{-1}{\sqrt[5]{4}\,x} \to (f\circ g)'(x) = \dfrac{1}{\sqrt[5]{4}\,x^2} = \dfrac{1}{\sqrt[5]{4}\sqrt[5]{9}} = \dfrac{1}{\sqrt[5]{36}}$

$\lim\limits_{x\to0}\dfrac{\left(\dfrac{-1+\sin x}{\sin x+1}\right)^2-1}{x} = \lim\limits_{x\to0}\dfrac{\left(\dfrac{\sin x-1}{\sin x+1}-1\right)\left(\dfrac{\sin x-1}{\sin x+1}+1\right)}{x}$ ۶

$= \lim\limits_{x\to0}\dfrac{\left(\dfrac{\sin x-1-\sin x-1}{\sin x+1}\right)\left(\dfrac{\sin x-1+\sin x+1}{\sin x+1}\right)}{x} = \lim\limits_{x\to0}\dfrac{\left(\dfrac{-2}{\sin x+1}\right)\left(\dfrac{2\sin x}{\sin x+1}\right)}{x}$

هم‌ارزی $\to \lim\limits_{x\to0}\dfrac{-4\cancel{x}}{(x+1)^2\cancel{(x)}} = \dfrac{-4}{(0+1)^2} = -4$ ✓



$f'(x) = -2x \to -2\alpha,\ 2\alpha \to -4\alpha^2=-1 \to 4\alpha^2=1$ ۷

$\to \alpha^2=\dfrac14 \to \alpha_1=\dfrac12$

$\alpha_2=-\dfrac12 \to y=d=-(\pm\tfrac12)^2-1$

$\to y=d=-\dfrac14-1=-\dfrac54 \xrightarrow{\text{فاصله تا مبدا}} \dfrac54$ ✓

$d: y=kx \xrightarrow{①} \text{مقادیر برابر} \quad kx = 8\sqrt{x}\cdot x^2+4\sqrt{x}$ ۸

$f(x)=2\sqrt{x}\,(4x^2+3) \xrightarrow[\text{مشتق برابر}]{②} \quad k=\dfrac{4x^2+3}{\sqrt{x}}+16\sqrt{x}\cdot x \xrightarrow{\times x}$

$kx=\dfrac{4x^3+3x}{\sqrt{x}}+16\sqrt{x}\cdot x^2 \xrightarrow{①=②} 8\sqrt{x}\cdot x^2+4\sqrt{x} = \dfrac{4x^3+3x}{\sqrt{x}}+16\sqrt{x}\cdot x^2 \xrightarrow{\times\sqrt{x}}$

$8x^3+4x = 4x^3+3x+16x^3 \to 12x^3-3x=0 \to 3x(4x^2-1)=0$

$\to x=0$ ✗ $\quad x=\pm\dfrac12$ ✓ $\to ① \ \dfrac{k}{2} = \dfrac{8\sqrt2}{2}\times\dfrac14+\dfrac{4\sqrt2}{2} \to \boxed{k=8\sqrt2}$ ✓ شیب

$d: y=kx \quad ①$ مقادیر برابر $\quad kx=\dfrac{\sqrt{x}}{-2x^2+x+1}$ ۹

$f(x)=\dfrac{\sqrt{x}}{-2x^2+x+1} \quad ②$ مشتق برابر $\quad k=\dfrac{\dfrac{-2x^2+x+1}{2\sqrt{x}}+4\sqrt{x}\cdot x-\sqrt{x}}{(-2x^2+x+1)^2} \xrightarrow{\times x} kx=\dfrac{\dfrac{-2x^3+x^2+x}{2\sqrt{x}}+4\sqrt{x}\cdot x^2-\sqrt{x}\cdot x}{(-2x^2+x+1)^2}$

$\xrightarrow{①=②} \sqrt{x} = \dfrac{\dfrac{-2x^3+x^2+x}{2\sqrt{x}}+4\sqrt{x}\cdot x^2-\sqrt{x}\cdot x}{(-2x^2+x+1)} \xrightarrow{\times2\sqrt{x}} -2x^3+x^2+x = \dfrac{-2x^3+x^2+x}{2}+\dfrac{4x^3-x^2}{1}$

$\xrightarrow[\text{ساده سازی}]{\times2} x(10x^2-3x-1)=0 \xrightarrow[x>0]{x\neq0} x=\dfrac{3+\sqrt{40+9}}{20}=\dfrac12 \xrightarrow{①} \dfrac{k}{2}=\dfrac{\frac{\sqrt2}{2}}{-2(\frac14)+\frac12+1} \to k=\sqrt2$

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$\to y=\dfrac12 x \xrightarrow{x=\sqrt2} \boxed{y=\dfrac{\sqrt2}{2}}$ ✓

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$$fog(x) = \left(\frac{1}{\sqrt{x^2-1}} \left[\frac{1}{\sqrt{x^2-1}} \right] \right)^3 \xrightarrow{x \to \frac{\sqrt{5}}{2}^-} x < \frac{\sqrt{5}}{2} \to x^2 < \frac{5}{4}$$

$$\to x^2 - 1 < \frac{1}{4} \to \sqrt{x^2-1} < \frac{1}{2} \to \frac{1}{\sqrt{x^2-1}} > 2 \to \left[\frac{1}{\sqrt{x^2-1}} \right] = 2$$

$$\to (fog)(x) = \left(\frac{2}{\sqrt{x^2-1}} \right)^3 \to (fog)'(x) = 3 \left(\frac{2}{\sqrt{x^2-1}} \right)^2 \left(\frac{-2x}{\sqrt{x^2-1}} \right)$$

$$\xrightarrow{x = \frac{\sqrt{5}}{2}} 3 \left(\frac{2}{\frac{1}{2}} \right)^2 \left(\frac{-2\left(\frac{\sqrt{5}}{2}\right)}{\frac{1}{2}} \right) = 3(16)(-2\sqrt{5}) = -48\sqrt{5} \times 3$$

$\to -48\sqrt{5}$ برابر ۳ ✓