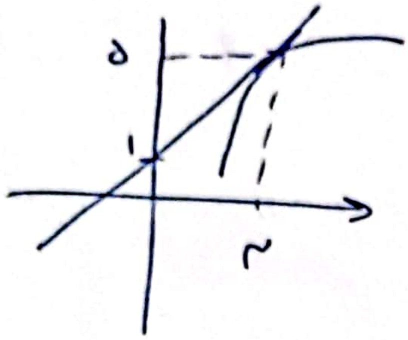
به نام خدا

سامان فرزادگر

تکلیف ۲۴

۱۲ پسر B

۱۹٫۵ آفرین خیلی عالی نوشتی!



۱ـ

معادله خط گذرا از نقاط : $(3,5)$ و $(0,1)$

$m = \frac{\Delta y}{\Delta x} = \frac{4}{3}$

$\Rightarrow y - 1 = m(x - 0)$

$\Rightarrow y = \frac{4}{3}x + 1$

$\Rightarrow f'(3) = m = \frac{4}{3}$

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$m = \frac{\Delta y}{\Delta x} = \frac{1}{3} \Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 1 = \frac{1}{3}(x+1) \Rightarrow \boxed{y = \frac{1}{3}x + \frac{4}{3}}$

معادله تلاقی باید ریشه مضاعف داشته باشد. $\Delta = 0$

$\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 3\sqrt{ax-1} = x + 4 \Rightarrow 9(ax-1) = x^2 + 16 + 8x$

$\Rightarrow 9ax - 9 = x^2 + 16 + 8x \Rightarrow x^2 + x(8 - 9a) + 25 = 0$

$\Rightarrow \Delta = 0 \Rightarrow (8 - 9a)^2 - 4 \times 25 = 0 \Rightarrow 8 - 9a = 10 \Rightarrow a = -\frac{2}{9}$

$8 - 9a = -10 \Rightarrow a = 2$

$\begin{cases} a = -\frac{2}{9} \Rightarrow x^2 + 10x + 25 = 0 \Rightarrow x = -5 \Rightarrow \frac{1}{3}x + \frac{4}{3} < 0 & \text{غ ق ق} \\ a = 2 \Rightarrow x^2 - 10x + 25 = 0 \Rightarrow x = 5 \end{cases}$

2

$\Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \sqrt{10-1} = 3$

۳ـ معادله تلاقی باید ریشه مضاعف (۱) داشته باشد

$4y - 3x = n \Rightarrow y = \frac{3}{4}x + \frac{n}{4}$

$\Rightarrow y_1 = y_2 \Rightarrow \frac{x^2 + mx + 1}{x + 3} = \frac{3}{4}x + \frac{n}{4} \Rightarrow x^2 + mx + 1 = \frac{3}{4}x^2 + \left(\frac{9}{4} + \frac{n}{4}\right)x + \frac{3n}{4}$

$\Rightarrow \frac{1}{4}x^2 + x\left(m - \frac{9}{4} - \frac{n}{4}\right) + 1 - \frac{3n}{4} = 0$

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$\Rightarrow \frac{-b}{2a} = \frac{-\left(m - \frac{9}{4} - \frac{n}{4}\right)}{\frac{1}{2}} = 1 \Rightarrow m - \frac{9}{4} - \frac{n}{4} = -\frac{1}{2}$

$\Rightarrow \Delta = 0 \Rightarrow \left(\underbrace{m - \frac{9}{4} - \frac{n}{4}}_{-\frac{1}{2}}\right)^2 - 4\left(\frac{1}{4}\right)\left(1 - \frac{3n}{4}\right) = 0$

$\Rightarrow \frac{1}{4} = 1 - \frac{3n}{4} \Rightarrow \boxed{n = 1} \Rightarrow m - \frac{10}{4} = -\frac{1}{2} \Rightarrow \boxed{m = 2}$

$m \cdot n = 2$ ✓ 2

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$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3 - \sin x)(9 + \sin^2 x + 3\sin x)}{(3 - \sin x)(3 + \sin x)}$

$= \frac{9 + \sin^2 x + 3\sin x}{3 + \sin x}$

$3g(x) = \frac{9}{3 + \sin x}$

$3g(x) - f(x) = \frac{9 - 9 - \sin^2 x - 3\sin x}{3 + \sin x} = \frac{-\sin x(\sin x + 3)}{3 + \sin x} = -\sin x$

$[3g(x) - f(x)]' = 3g'(x) - f'(x) = (-\sin x)' = -\cos x$

$x = \frac{5\pi}{3}$

$\Rightarrow 3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$ 2

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$g'(\sqrt[5]{2})\, f'(g(\sqrt[5]{2})) = (f \circ g)'(\underset{x>0}{\sqrt[5]{2}})$

$f \circ g(x) = \frac{-1}{\sqrt[5]{2g}} = \frac{-1}{\sqrt[5]{\frac{2}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x$

$x > 0$

$\hookrightarrow g(x) = \frac{1}{2x^5} > 0$ ✓

$\Rightarrow (f \circ g)'(x) = -1$ 2

$\Rightarrow (f \circ g)'(\sqrt[5]{2}) = -1$

$$f(x) = x\,g(x) + 1 \Rightarrow g(x) = \frac{f(x) - 1}{x} = \frac{\left(\frac{\sin x - 1}{\sin x + 1}\right)^2 - 1}{x}$$

$$\lim_{x \to 0} g(x) = \frac{\left(\frac{\sin x - 1}{\sin x + 1}\right)^2 - 1}{x} \overset{\text{هم ارزی}}{=} \lim_{x \to 0} \frac{\left(\frac{x - 1}{x + 1}\right)^2 - 1}{x}$$

$$= \lim_{x \to 0} \frac{\frac{x^2 - 2x + 1}{x^2 + 1 + 2x} - 1}{x} = \frac{\frac{-4x}{x^2 + 1 + 2x}}{x} = \frac{-4}{x^2 + 1 + 2x} \overset{x \to 0}{=} \frac{-4}{1} = -4$$

حل با هوپیتال :

$$\lim_{x \to 0} \frac{\left(\frac{\sin x - 1}{\sin x + 1}\right)^2 - 1}{x} \xrightarrow{HP} \frac{2\left(\frac{\sin x - 1}{\sin x + 1}\right)\left(\frac{\cos x(\sin x + 1) - \cos x(\sin x - 1)}{(\sin x + 1)^2}\right)}{1}$$

$$x \to 0:\quad = \frac{2(-1)\left(\frac{1 + 1}{1}\right)}{1} = -4$$

$$y = x^2 + 1 \xrightarrow[\text{محور x ها}]{\text{قرینه نسبت به}} y_1 = -x^2 - 1$$

$$y_2 = d$$

$$\Rightarrow -x^2 - 1 = d \Rightarrow x^2 + d + 1 = 0 \quad \text{I}$$

$$y_1' = -2x \Rightarrow \begin{cases} -2\alpha & \text{شیب مماس اول} \\ -2\beta & \text{شیب مماس دوم} \end{cases} \Rightarrow (-2\alpha)(-2\beta) = -1 \Rightarrow 4\alpha\beta = -1 \Rightarrow \alpha\beta = -\frac{1}{4}$$

چون α و β ریشه‌های معادله I هستند پس ضرب ریشه‌ها $\left(-\frac{1}{4}\right)$ است

$$\Rightarrow x^2 + d + 1 = 0 \Rightarrow \alpha\beta = \frac{c}{a} = d + 1 = -\frac{1}{4} \Rightarrow d = -\frac{5}{4}$$

فاصله خط از مبدأ مختصات $= \frac{5}{4}$

۸_ معادله خط مبدأ گذر : $y = Ax$

معادله تلاقی :

$$f(x) = y \Rightarrow 2\sqrt{x}(4x^2 + 3) = Ax$$

$$\Rightarrow 2(4x^2 + 3) = A\sqrt{x} \Rightarrow 8x^2 + 6 = A\sqrt{x}$$

چون معادله ریشه مضاعف دارد پس مشتق آن هم همان ریشه را دارد از ۲ طرف مشتق می‌گیریم:

$$8x^2+6=A\sqrt{x} \xrightarrow{\text{مشتق}} 16x=\frac{A}{2\sqrt{x}} \Rightarrow A=32x\sqrt{x}$$

در معادله اول مساوی داریم

$$\Rightarrow 8x^2+6=(32x\sqrt{x})\times\sqrt{x} \Rightarrow 8x^2+6=32x^2 \Rightarrow 24x^2=6$$

$$\Rightarrow x^2=\frac{1}{4} \Rightarrow x=\frac{1}{2} \Rightarrow \begin{cases} A=32x\sqrt{x} \\ x=\frac{1}{2} \end{cases} A=16\frac{\sqrt{2}}{2}=\boxed{8\sqrt{2}}$$

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۹- معادله خط مبدأ گذر: $y=ax$

$$f(x)=\frac{\sqrt{x}}{-2x^2+x+1}$$ معادله تلاقی: $$\frac{\sqrt{x}}{-2x^2+x+1}=ax \Rightarrow a\sqrt{x}=\frac{1}{-2x^2+x+1}$$

چون معادله ریشه مضاعف دارد پس مشتق آن هم ریشه را دارد:

$$\Rightarrow -2x^2\sqrt{x}+x\sqrt{x}+\sqrt{x}=\frac{1}{a}$$

$$-2x^{2.5}+x^{1.5}+x^{0.5}=\frac{1}{a} \xrightarrow{\text{مشتق}} -5x^{1.5}+1.5x^{0.5}+0.5x^{-0.5}=0$$

$$\xrightarrow{\times\sqrt{x}} -5x^2+1.5x+0.5=0 \xrightarrow{\times 2} -10x^2+3x+1=0 \Rightarrow \begin{cases} x=\frac{1}{2} \\ x=-\frac{1}{5} \text{ غ ق} \end{cases}$$

$x>$

$$\Rightarrow f\left(\tfrac{1}{2}\right)=\frac{\sqrt{\frac{1}{2}}}{1}=\frac{\sqrt{2}}{2}$$ ب

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$$(f\circ g)'(x)=g'(x)\,f'(g(x))$$

$$g(x)=\frac{1}{\sqrt{x^2-1}} \Rightarrow g'(x)=\frac{-2x}{2\sqrt{(x^2-1)^3}}=\frac{-x}{\sqrt{(x^2-1)^3}} \; ; \; g\left(\tfrac{\sqrt{5}}{2}\right)=2$$

$$f(x)=(x[x])^2=x^2 \Rightarrow f'(x)=2x$$

$$x=\frac{\sqrt{5}}{2}\simeq 1.1$$

$$\Rightarrow g'(x)\, f'(g(x)) = \frac{-x}{\sqrt{5-x^2}} \times 3g^2(x)$$

1,5

$$\xrightarrow{x \to \frac{\sqrt{5}}{2}} \frac{-\frac{\sqrt{5}}{2}}{\frac{1}{8}} \times 3 \times 4 = -48\sqrt{5}$$

$$\Rightarrow \frac{\text{طرف}}{-48\sqrt{5}} = \frac{-48\sqrt{5}}{-48\sqrt{5}} = 1$$

برابر است.

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$$g(x) = (x^2-1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$$

$$g'\left(\frac{\sqrt{5}}{2}\right) = -\frac{1}{2}(\sqrt{5})\left(\frac{5}{4}-1\right)^{-\frac{3}{2}} \longrightarrow -\frac{\sqrt{5}}{2}\left(\underset{8}{\cancel{2^{-2\left(-\frac{3}{2}\right)}}}\right) = -4\sqrt{5}$$

$$g\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{\frac{5}{4}^- - 1}} = \frac{1}{\sqrt{\frac{1}{4}^-}} = \frac{1}{\frac{1}{2}^-} = 2^+$$

$$f'(2^+) = \left((2x)^3\right)' = 24x^2 = 24\times 4$$

$$(f\circ g)'\left(\frac{\sqrt{5}}{2}^-\right) = -4\sqrt{5}\times 24\times 4 \xrightarrow{\div -48\sqrt{5}} \frac{\overset{2}{\cancel{24}}\times\cancel{4}\times -4\sqrt{5}}{-\cancel{48}\sqrt{5}} = 8$$