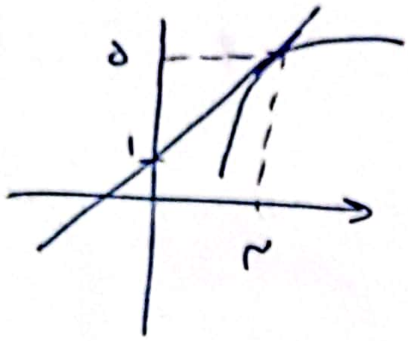
بنام خدا

سامان فرازگر

۱۲ پسر B

تکلیف ۲۴



۱-

معادله خط گذرا از نقاط (۳,۵) و (۰,۱):

$$m = \frac{\Delta y}{\Delta x} = \frac{4}{3}$$

$$\Rightarrow y - 1 = m(x - 0)$$

$$\Rightarrow y = \frac{4}{3}x + 1$$

$$\Rightarrow f'(3) = m = \frac{4}{3}$$

۲-

$$m = \frac{\Delta y}{\Delta x} = \frac{1}{3} \Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 1 = \frac{1}{3}(x+1) \Rightarrow \boxed{y = \frac{1}{3}x + \frac{4}{3}}$$

معادله تلاقی باید ریشه مضاعف داشته باشد. $\underbrace{\quad}_{\Delta = 0}$

$$\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 3\sqrt{ax-1} = x + 4 \Rightarrow 9(ax-1) = x^2 + 16 + 8x$$

$$\Rightarrow 9ax - 9 = x^2 + 16 + 8x \Rightarrow x^2 + x(8 - 9a) + 25 = 0$$

$$\Rightarrow \Delta = 0 \Rightarrow (8 - 9a)^2 - 4 \times 25 = 0 \Rightarrow 8 - 9a = 10 \Rightarrow a = -\frac{2}{9}$$

$$8 - 9a = -10 \Rightarrow a = 2$$

$$\begin{cases} a = -\frac{2}{9} \Rightarrow x^2 + 10x + 25 = 0 \Rightarrow x = -5 \Rightarrow \frac{1}{3}x + \frac{4}{3} < 0 \quad \text{غ ق ق} \\ a = 2 \Rightarrow x^2 - 10x + 25 = 0 \Rightarrow x = 5 \checkmark \end{cases}$$

$$\Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \sqrt{10-1} = 3$$

۳- معادله تلاقی باید ریشه مضاعف (۱) داشته باشد

$$4y - 3x = n \Rightarrow y = \frac{3}{4}x + \frac{n}{4}$$

$$\Rightarrow y_1 = y_2 \Rightarrow \frac{x^2 + mx + 1}{x + 3} = \frac{3}{4}x + \frac{n}{4} \Rightarrow x^2 + mx + 1 = \frac{3}{4}x^2 + \left(\frac{9}{4} + \frac{n}{4}\right)x + \frac{3n}{4}$$

$$\Rightarrow \frac{1}{4}x^2 + x\left(m-\frac{9}{4}-\frac{n}{4}\right) + 1 - \frac{3n}{4} = 0$$

از شرایط

$$\Rightarrow \frac{-b}{2a} = \frac{-\left(m-\frac{9}{4}-\frac{n}{4}\right)}{\frac{1}{2}} = 1 \Rightarrow m-\frac{9}{4}-\frac{n}{4} = -\frac{1}{2}$$

$$\Rightarrow \Delta = 0 \Rightarrow \underbrace{\left(m-\frac{9}{4}-\frac{n}{4}\right)}_{-\frac{1}{2}}^{2} - 4\left(\frac{1}{4}\right)\left(1-\frac{3n}{4}\right) = 0$$

$$\Rightarrow \frac{1}{4} = 1 - \frac{3n}{4} \Rightarrow \boxed{n=1} \Rightarrow m - \frac{10}{4} = -\frac{1}{2} \Rightarrow \boxed{m=2}$$

$$m \cdot n = 2$$

-۴

$$f(x) = \frac{27-\sin^3 x}{9-\sin^2 x} = \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)}$$

$$= \frac{9+\sin^2 x+3\sin x}{3+\sin x}$$

$$3g(x) = \frac{9}{3+\sin x}$$

$$3g(x) - f(x) = \frac{9-9-\sin^2 x-3\sin x}{3+\sin x} = \frac{-\sin x(\sin x+3)}{3+\sin x} = -\sin x$$

$$[3g(x)-f(x)]' = 3g'(x) - f'(x) = (-\sin x)' = -\cos x$$

$$x = \frac{5\pi}{3}$$

$$\Rightarrow 3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$$

-۵

$$g'(\sqrt[5]{2})\,f'(g(\sqrt[5]{2})) = (f\circ g)'(\sqrt[5]{2})$$

$x>0$

$$f\circ g(x) = \frac{-1}{\sqrt[5]{2g}} = \frac{-1}{\sqrt[5]{\frac{2}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x$$

$x>0$

$$\hookrightarrow g(x) = \frac{1}{2x^5}, \quad x>0$$

$$\Rightarrow (f\circ g)'(x) = -1$$

$$\Rightarrow (f\circ g)'(\sqrt[5]{2}) = -1$$

$$f(x) = x\,g(x) + 1 \Rightarrow g(x) = \frac{f(x)-1}{x} = \frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2 - 1}{x}$$

$$\lim_{x\to 0} g(x) = \frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2 - 1}{x} \overset{\text{هم ارزی}}{=} \lim_{x\to 0} \frac{\left(\frac{x-1}{x+1}\right)^2 - 1}{x}$$

$$= \lim_{x\to 0} \frac{\frac{x^2-2x+1}{x^2+1+2x} - 1}{x} = \frac{\frac{-4x}{x^2+1+2x}}{x} = \frac{-4}{x^2+1+2x} \overset{x\to 0}{=} \frac{-4}{1} = -4$$

حل با هوپیتال :

$$\lim_{x\to 0} \frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2 - 1}{x} \xrightarrow{HP} \frac{2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{\cos x(\sin x+1) - \cos x(\sin x-1)}{(\sin x+1)^2}\right)}{1}$$

$$\overset{x\to 0}{=} \frac{2(-1)\left(\frac{1+1}{1}\right)}{1} = -4$$

$$y = x^2+1 \xrightarrow[\text{محور x ها}]{\text{قرینه نسبت به}} y_1 = -x^2-1$$

$$y_2 = d$$

۷_

$$\Rightarrow -x^2-1 = d \Rightarrow \boxed{x^2+d+1=0} \quad I$$

$$y_1' = -2x \Rightarrow \begin{cases} -2\alpha & \text{شیب مماس اول} \\ -2\beta & \text{شیب مماس دوم} \end{cases} \Rightarrow (-2\alpha)(-2\beta) = 4\alpha\beta = -1 \Rightarrow \alpha\beta = -\frac{1}{4}$$

چون α و β ریشه‌های معادله I هستند پس ضرب ریشه‌ها $\left(-\frac{1}{4}\right)$ است

$$\Rightarrow x^2+d+1=0 \Rightarrow \alpha\beta = \frac{c}{a} = d+1 = -\frac{1}{4} \Rightarrow d = -\frac{5}{4}$$

فاصله خط از مبدا مختصات $= \frac{5}{4}$

۸_ معادله خط مبدا گذر :

$$y = Ax$$

معادله تلاقی :

$$f(x) = y \Rightarrow 2\sqrt{x}(4x^2+3) = Ax$$

$$\Rightarrow 2(4x^2+3) = A\sqrt{x} \Rightarrow 8x^2+6 = A\sqrt{x}$$

چون معادله ریشه مضاعف دارد پس مشتق آن هم همان ریشه را دارد از ۲ طرف مشتق می‌گیریم:

$$8x^2+6=A\sqrt{x} \xrightarrow{\text{مشتق}} 16x=\frac{A}{2\sqrt{x}} \Rightarrow A=32x\sqrt{x}$$

در معادله اول می‌گذاریم

$$\Rightarrow 8x^2+6=(32x\sqrt{x})\times\sqrt{x} \Rightarrow 8x^2+6=32x^2 \Rightarrow 24x^2=6$$

$$\Rightarrow x^2=\frac{1}{4} \Rightarrow x=\frac{1}{2} \Rightarrow \left.\begin{array}{l} A=32x\sqrt{x} \\ x=\frac{1}{2} \end{array}\right\} A=16\frac{\sqrt{2}}{2}=\boxed{8\sqrt{2}}$$

۹- معادله خط مبدأ گذر: $y=ax$

$$f(x)=\frac{\sqrt{x}}{-2x^2+x+1}$$ معادله تلاقی: $$\frac{\sqrt{x}}{-2x^2+x+1}=ax \Rightarrow a\sqrt{x}=\frac{1}{-2x^2+x+1}$$

$$\Rightarrow -2x^2\sqrt{x}+x\sqrt{x}+\sqrt{x}=\frac{1}{a}$$ چون معادله ریشه مضاعف دارد پس مشتق آن هم ریشه را دارد:

$$-2x^{2.5}+x^{1.5}+x^{0.5}=\frac{1}{a} \xrightarrow{\text{مشتق}} -5x^{1.5}+1.5x^{0.5}+0.5x^{-0.5}=0$$ همان

$$\xrightarrow{\times\sqrt{x}} -5x^2+1.5x+0.5=0 \xrightarrow{\times 2} -10x^2+3x+1=0 \Rightarrow \begin{cases} x=\frac{1}{2} \\ x=-\frac{1}{5} \text{ غق} \end{cases}$$

$x>$

$$\Rightarrow f\left(\frac{1}{2}\right)=\frac{\sqrt{\frac{1}{2}}}{1}=\frac{\sqrt{2}}{2}$$

۱۰- $(f\circ g)'(x)=g'(x)\,f'(g(x))$

$$g(x)=\frac{1}{\sqrt{x^2-1}} \Rightarrow g'(x)=\frac{-2x}{2\sqrt{(x^2-1)^3}}=\frac{-x}{\sqrt{(x^2-1)^3}} \; ; \; g\left(\frac{\sqrt{5}}{2}\right)=2$$

$$f(x)=(x[x])^2=x^2 \Rightarrow f'(x)=2x$$

$$x=\frac{\sqrt{5}}{2}\simeq 1.1$$

$$\Rightarrow g'(x)\, f'(g(x)) = \frac{-x}{\sqrt{5-4x^2}} \times 3g^2(x)$$

$$\xrightarrow{x \to \frac{\sqrt{5}}{2}^-} \frac{-\frac{\sqrt{5}}{2}}{\frac{1}{8}} \times 3 \times 4 = -48\sqrt{5}$$

$$\Rightarrow \frac{\text{حاصل}}{-48\sqrt{5}} = \frac{-48\sqrt{5}}{-48\sqrt{5}} = 1$$ برابر است.