Date :

«مهرگان کرد سیانی» تالیف ۲۴

۱۸٫۷۵

Page : ( )

(1) شیب خط مماس در نقطه $x=3$ بر نمودار مساوی $f'(3)$ است پس شیب خط مماس را بدست آوریم

دقت کن :: $\frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$

$f'(3) = \frac{4}{3} = 2$

۱٫۲۵

(2) $m_{خط} = \frac{1}{3} \Rightarrow f'(a) = \frac{1}{3} = \frac{a}{2\sqrt{2x-1}} \Rightarrow x = \frac{9a^2+4}{4a}$

$\Rightarrow \frac{1}{3}x + \frac{4}{3} = y \Rightarrow \frac{1}{3}\left(\frac{9a^2+4}{4a}\right) + \frac{4}{3} = \sqrt{\frac{9a^2+4}{2} - 1}$

$\Rightarrow \frac{-9a^2+16a+4}{12a} = 0 \Rightarrow a = 2 , -\frac{2}{9} \times \Rightarrow x = 5$

$\Rightarrow f(5) = \sqrt{2x-1} = \sqrt{9} = 3$

(3) $y = \frac{3}{4}x + \frac{n}{4} \Rightarrow f'(1) = \frac{3}{4} = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{6+3m}{16}$

$\Rightarrow m = 2 \Rightarrow f(x) = \frac{x^2+2x+1}{x+3} \Rightarrow f(1) = 1 = \frac{3}{4} + \frac{n}{4} \Rightarrow \boxed{n=1}$

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$m + n = 3$

(4) $f(x) = \frac{\cancel{(3-\sin x)}(\sin^2 + 3\sin x + 9)}{\cancel{(3-\sin x)}(3+\sin x)} = \frac{\sin^2 + 3\sin x + 9}{3+\sin x}$

$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = (3g-f)'\left(\frac{5\pi}{3}\right) \Rightarrow \frac{3\sin^2 x + 9\sin x + 24}{3+\sin x} = 3g - f$

۱٫۵

$(3g-f)'(x) = \frac{(6\sin x\cos x)(3+\sin x) - (\cos x)(3\sin^2 x + 9\sin x + 24)}{(3+\sin x)^2}$

$\Rightarrow \frac{\left(6 \times \frac{-\sqrt{3}}{2} \times \frac{1}{2}\right)\left(3 - \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{9}{4} - \frac{9\sqrt{3}}{2} + 24\right)}{\left(3 - \frac{\sqrt{3}}{2}\right)^2}$

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$(f\circ g)'(\sqrt[5]{3}) = ? \Rightarrow g(x) = \frac{1}{2x^5} \quad , \quad f(x) = \frac{1}{\sqrt[5]{2x}}$

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$\Rightarrow f\circ g(x) = \frac{1}{\sqrt[5]{2\,\frac{1}{2x^5}}} = x \Rightarrow (f\circ g)'(\sqrt[5]{3}) = \underline{1}$

6

$g(x) = \frac{f(x)-1}{x} \Rightarrow \frac{\cancel{\sin^2 x} - 2\sin x \cancel{+1} - \cancel{\sin^2 x} \cancel{-1} - 2\sin x}{1+\sin^2 x + 2\sin x}$

$g(x) = \frac{\frac{-4\sin x}{(1+\sin x)^2}}{x} \Rightarrow \lim_{x\to 0} g(x) = \frac{-4x}{(1+x)^2 x} = \frac{-4}{(1+x)^2} = -4$

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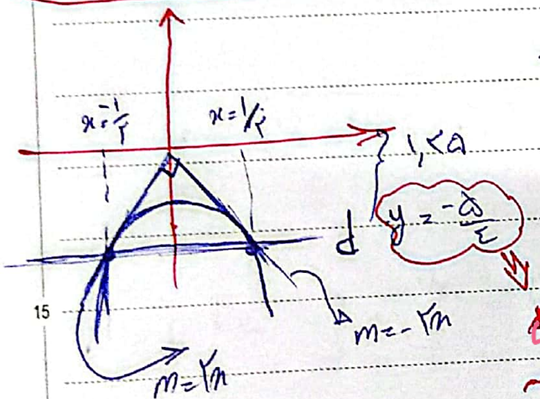
7

$-2x = \frac{-1}{2x} \rightarrow 4x^2 = 1 \rightarrow x^2 = \frac{1}{4}, x = \pm\frac{1}{2}$



$y = -x^2 - 1 \rightarrow y' = -2x$

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غلط

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$y = ax \Rightarrow f'(x) = \frac{1}{\sqrt{x}}(4x^2+3) + 8x^2(2\sqrt{x}) = a$

$2\sqrt{x}(4x^2+3) = ax \Rightarrow \cancel{\sqrt{x}}(4x^2+3) + 8x^2(2\cancel{\sqrt{x}}) = 2\cancel{\sqrt{x}}(4x^2+3)$

$\Rightarrow 12x^2 - 3 = 0 \Rightarrow x = \begin{cases} -\frac{1}{2} \\ \frac{1}{2} \end{cases} \Rightarrow a = \frac{2\sqrt{\frac{1}{2}} \times 4}{\frac{1}{2}} = \frac{16}{\sqrt{2}} = 8\sqrt{2}$

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$$m_d = \frac{f(x)-f(0)}{x-0} = \frac{\sqrt{x}}{(-2x^2+x+1)\,x}$$

$$f'(x) = \frac{\frac{-2x^2+x+1}{2\sqrt{x}} - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} \Rightarrow \frac{\sqrt{x}}{-2x^2+x} = \frac{6x^2-x+1}{2\sqrt{x}\,(-2x^2+x+1)}$$

$$= \frac{-2x^2+x+1+8x^2-2x}{2\sqrt{x}} = \frac{6x^2-x+1}{2\sqrt{x}}$$

$$(6x^2-x+1) = 6x^2+2x+2$$

2

$$10x^2-3x-1=0 \Rightarrow \frac{3\pm 7}{20} \begin{cases} \frac{1}{2} \\ -\frac{1}{5} \end{cases}$$

$$\Rightarrow f(\tfrac{1}{2}) = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{2}+\frac{1}{2}+1} = \frac{\sqrt{2}}{2}$$

10

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$$(f\circ g)' = \underbrace{g'(\tfrac{\sqrt{5}}{2})}\,\underbrace{f'(g(\tfrac{\sqrt{5}}{2}))} \Rightarrow \sqrt{5} \times 12 = 12\sqrt{5}$$

$$g'(x) = \frac{2x}{2\sqrt{x^2-1}} \Rightarrow g'(\tfrac{\sqrt{5}}{2}) = \sqrt{5}$$

$$f'(x) = 3x^2 \Rightarrow f'(2) = 12$$

$$g(\tfrac{\sqrt{5}}{2}) = 2$$

$$\Rightarrow \frac{12\sqrt{5}}{-48\sqrt{5}} = -4$$

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$$۳g - f(x) = \frac{۹}{۳+\sin x} - \frac{(۳-\cancel{\sin x})(۹+\sin^{۲} x+۳\sin x)}{(۳-\cancel{\sin x})(۳+\sin x)} = \frac{-\sin x(\cancel{\sin x}+۳)}{\cancel{\sin x}+۳}$$

با $-\sin x \xrightarrow{\text{مشتق}} (۳g - f)'(x) = -\cos x \rightsquigarrow -\cos\left(\frac{۵\pi}{۳}\right) = \boxed{-\frac{۱}{۲}}$

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$$g(x) = (x^{۲}-۱)^{-\frac{۱}{۲}} \longrightarrow g'(x) = -\frac{۱}{۲}(۲x)(x^{۲}-۱)^{-\frac{۳}{۲}}$$

$$g'\left(\frac{\sqrt{۵}}{۲}\right) = -\frac{۱}{۲}(\sqrt{۵})\left(\frac{۵}{۴}-۱\right)^{-\frac{۳}{۲}} \longrightarrow -\frac{\sqrt{۵}}{۲}\left(\underbrace{\cancel{۲^{-۲(-\frac{۳}{۲})}}}_{۸}\right) = -۴\sqrt{۵}$$

$$g\left(\frac{\sqrt{۵}}{۲}^{-}\right) = \frac{۱}{\sqrt{\frac{۵^{-}}{۴}-۱}} = \frac{۱}{\sqrt{\frac{۱}{۴}^{-}}} = \frac{۱}{\frac{۱}{۲}^{-}} = ۲^{+}$$

$$f'(۲^{+}) = ((۲x)^{۳})' = ۲۴x^{۲} = ۲۴\times ۴$$

$$fog'\left(\frac{\sqrt{۵}^{-}}{۲}\right) = -۴\sqrt{۵}\times ۲۴\times ۴ \xrightarrow{\div -۴۸\sqrt{۵}} \frac{\overset{۲}{\cancel{۲۴}}\times\cancel{۴}\times -۴\sqrt{۵}}{-\cancel{۴۸}\sqrt{۵}} = ۸$$

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$$g'(x)\times f'(g(x)) = (fog)'(x)$$

$$x>0 \longrightarrow g(x) = \frac{۱}{۲x^{۵}} \longrightarrow f(x) = \frac{-۱}{\sqrt[۵]{۲x}} \rightsquigarrow fog(x) = \frac{-۱}{\sqrt[۵]{۲\left(\frac{۱}{۲x^{۵}}\right)}}$$

$$fog(x) = -x \longrightarrow fog'(x) = \boxed{-۱} \rightsquigarrow fog'(\sqrt{۳}) = ۱$$