«مهرو کرد سپیانی» تالیف ۲۴

(1) شیب خط مماس در نقطه $x=3$ بر نمودار مساوی $f'(3)$ است پس شیب خط مماس را بدست آوریم

$$f'(3) = \frac{4}{2} = 2$$

(2)

$$m = \frac{1}{3} \Rightarrow f'(A) = \frac{1}{3} = \frac{a}{2\sqrt{2x-1}} \Rightarrow x = \frac{9a^2+4}{8a}$$

$$\frac{1}{3}x + \frac{4}{3} = y \Rightarrow \frac{1}{3}\left(\frac{9a^2+4}{8a}\right) + \frac{4}{3} = \sqrt{\frac{9a^2+4}{4} - 1}$$

$$\Rightarrow \frac{-9a^2+16a+4}{12a} = 0 \Rightarrow a = 2 \quad \left(-\frac{2}{9} \times\right) \Rightarrow x = 5$$

$$\Rightarrow f(5) = \sqrt{2x-1} = \sqrt{9} = 3$$

(3)

$$y = \frac{3}{4}x + \frac{n}{4} \Rightarrow f'(1) = \frac{3}{4} = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{6+3m}{16}$$

$$\Rightarrow m = 2 \Rightarrow f(x) = \frac{x^2+2x+1}{x+3} \Rightarrow f(1) = 1 = \frac{3}{4} + \frac{n}{4} \Rightarrow n = 1$$

$$m + n = 3$$

(4)

$$f(x) = \frac{(3-\sin x)(\sin^2 x + 3\sin x + 9)}{(3-\sin x)(3+\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$$

$$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = (3g-f)'\left(\frac{5\pi}{3}\right) \Rightarrow \frac{3\sin^2 x + 9\sin x + 24}{3+\sin x} = 3g - f$$

$$(3g-f)'(x) = \frac{(6\sin x\cos x)(3+\sin x) - (\cos x)(3\sin^2 x + 9\sin x + 24)}{(3+\sin x)^2}$$

$$\Rightarrow \frac{\left(6 \times \frac{-\sqrt{3}}{2} \times \frac{1}{2}\right)\left(3 - \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{9}{4} - \frac{9\sqrt{3}}{2} + 24\right)}{\left(3 - \frac{\sqrt{3}}{2}\right)^2}$$

5

$(f\circ g)'(\sqrt[5]{3}) = ?$ ⇒ $g(x) = \frac{1}{3x^5}$, $f(x) = \frac{1}{\sqrt[5]{3x}}$

⇒ $f\circ g(x) = \frac{1}{\sqrt[5]{3\cdot\frac{1}{3x^5}}} = x$ ⇒ $(f\circ g)'(\sqrt[5]{3}) = 1$

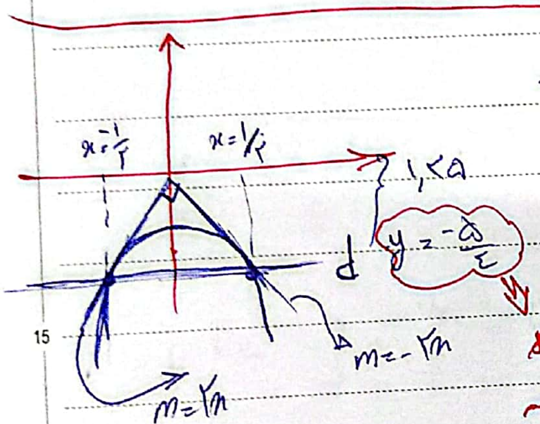
6

$g(x) = \frac{f(x)-1}{x}$ ⇒ $\frac{\sin^2 x - 2\sin x + 1 - \sin^2 x - 1 - 2\sin x}{1+\sin^2 x + 2\sin x}$

$g(x) = \frac{\frac{-4\sin x}{(1+\sin x)^2}}{x}$ ⇒ $\lim_{x\to 0} g(x) = \frac{-4x}{(1+x)^2 x} = \frac{-4}{(1+x)^2} = -4$

7

$-2x = \frac{-1}{2x} \to 4x^2 = 1 \to x^2 = \frac{1}{4}, x = \pm\frac{1}{2}$



$y = -x^2 - 1 \to y' = -2x$

8

$y = ax$ ⇒ $f'(x) = \frac{1}{\sqrt{x}}(4x^2+3) + 8x(2\sqrt{x}) = a$

$2\sqrt{x}(4x^2+3) = ax$ ⇒ $\sqrt{x}(4x^2+3) + 8x^2(2\sqrt{x}) = 2\sqrt{x}(4x^2+3)$

⇒ $12x^2 - 3 = 0$ ⇒ $x = -\frac{1}{2}, \frac{1}{2}$ ✓ ⇒ $a = \frac{2\sqrt{\frac{1}{2}}\times 4}{\frac{1}{2}} = \frac{16}{\sqrt{2}} = 8\sqrt{2}$

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(9)

$$m_d = \frac{f(x)-f(0)}{x-0} = \frac{\sqrt{x}}{(-2x^2+x+1)\,x}$$

$$f'(x) = \frac{\frac{-2x^2+x+1}{2\sqrt{x}} - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} \Rightarrow \frac{\sqrt{x}}{-2x^3+x} = \frac{4x^2-x+1}{2\sqrt{x}\,(-2x^2+x+1)}$$

$$= \frac{-2x^2+x+1+8x^2-2x}{2\sqrt{x}} = \frac{6x^2-x+1}{2\sqrt{x}}$$

$$(6x^2-x+1) = 4x^2+2x+2$$

$$10x^2-3x-1=0 \Rightarrow \frac{3\pm7}{20} \left\langle \begin{array}{l} \frac{1}{2} \\ \frac{-1}{5} \end{array}\right.$$

$$\Rightarrow f(\tfrac{1}{2}) \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{2}+\frac{1}{2}+1} = \frac{\sqrt{2}}{2}$$

(10)

$$(f\circ g)' = g'\left(\tfrac{\sqrt{5}}{2}\right) f'\left(g\left(\tfrac{\sqrt{5}}{2}\right)\right) \Rightarrow \sqrt{5} \times 12 = 12\sqrt{5}$$

$$g'(x) = \frac{2x}{2\sqrt{x^2-1}} \Rightarrow g'\left(\tfrac{\sqrt{5}}{2}\right) = \sqrt{5} \qquad f'(x) = 3x^2 \Rightarrow f'(2) = 12$$

$$g\left(\tfrac{\sqrt{5}}{2}\right) = 2$$

$$\Rightarrow \frac{12\sqrt{5}}{-48\sqrt{5}} = -\frac{1}{4}$$