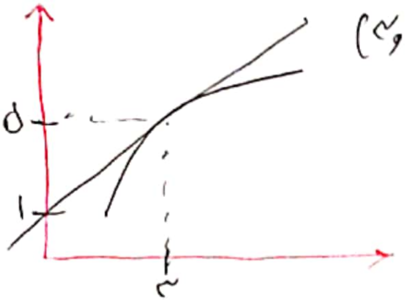

(۱)

$(۳,۵)$

$y-1=f'(۳)\times x \to ۵-1=f'(۳)\times ۳ \to ۳f'(۳)=۴ \to f'(۳)=\frac{۴}{۳}$

$\frac{۴}{۳}$

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(۲)

$f(x)=\sqrt{ax-1}$ $(-1,1)$ $(۲,۲)$ $f(۵)=?$

$m=\frac{۲-1}{۲+1}=\frac{1}{۳}$ $y=\frac{1}{۳}x+b \to ۲=\frac{1}{۳}\times ۲+b \to b=\frac{۴}{۳} \to y=\frac{1}{۳}x+\frac{۴}{۳}$

$\frac{1}{۳}x+\frac{۴}{۳}=\sqrt{ax-1} \to x+۴=۳\sqrt{ax-1} \to x^2+۱۶+۸x=۹(ax-1)=۹ax-۹$

$\to x^2+(۸-۹a)x+۲۵=0 \to \Delta=b^2-4ac=0 \to ۶۴+۸۱a^2-۱۴۴a-۱۰۰=0$

$\to ۸۱a^2-۱۴۴a-۳۶=0 \to a^2-\frac{۱۶}{۹}a-\frac{۴}{۹}=0 \to (a-\frac{۱۸}{۹})(a+\frac{۲}{۹})=0 \to a=۲, \cancel{-\frac{۲}{۹}}$

$f(x)=\sqrt{۲x-1}=\sqrt{۱۰-1}=\sqrt{۹}=۳$ $f(۵)=۳$

---

(۳)

$y=\frac{x^2+mx+1}{x+۳}$ $۴y-۳x=n$ $m+n=?$

$۴y-۳x+n \to m=\frac{۳}{۴}$

$\frac{x^2+۶x+۳mx-1}{(x+۳)^2} = \frac{۶+۳m}{۱۶}=\frac{۳}{۴} \to ۶+۳m=۱۲ \to m=۲$ ($x=1$) $m+n=1$

$f(1)=1 \to ۴(1)-۳(1)=n \to n-1$

---

(۴)

$f(x)=\frac{۲۷-\sin^3x}{۹-\sin^2x}$, $g(x)=\frac{۳}{۳+\sin x}$ $۳g'(\frac{۵\pi}{۳})-f'(\frac{۵\pi}{۳})$

$\frac{(۳-\sin)(\sin^2+۳\sin+۹)}{(۳-\sin)(۳+\sin)}$ $\frac{۹-(\sin^2+۳\sin+۹)}{۳+\sin}=\frac{-\sin^2-۳\sin}{۳+\sin}=\frac{-\sin(\sin+۳)}{۳+\sin}=-\sin$

$\xrightarrow{\text{مشتق}} -\cos \to -\cos(\frac{۵\pi}{۳})=-\frac{1}{۲}$

$-\frac{1}{۲}$

---

(۵)

$f(x)=-\frac{1}{\sqrt[۵]{x+|x|}}$, $g(x)=\frac{1}{x^۵+|x^۵|}$ $g'(\sqrt[۵]{۲})f'(g(\sqrt[۵]{۲}))=?$

$(f(g(x)))'$

$\sqrt[۵]{۲} \to g(x)=\frac{1}{۲x^۵} \Rightarrow -\frac{1}{\sqrt[۵]{۲x^۵}}=-x \xrightarrow{\text{مشتق}} -1$

-1

(6)

$f(x) = \left(\frac{-1+\sin x}{1+\sin x}\right)^2 \qquad f(x) = x g(x) + 1 \qquad \lim_{x \to 0} g(x) = ?$

$f'(x) = 2\left(\frac{\cos + \cos}{(\sin+1)^2}\right)\left(\frac{\sin - 1}{\sin + 1}\right) = 2 \times -2 = -4$

$\frac{f(x) - f(0)}{x - 0} = \frac{x g(x)}{x} = g(x)$

-4

$g(x) = -4$

---

(7)

$y = x^2 + 1 \to y' = 2x \to -4x^2 = -1 \to x^2 = \frac{1}{4} \to x = \pm\frac{1}{2}$

$m_1 - m$

$\left(\pm\frac{1}{2}\right)^2 + 1 = \frac{5}{4}$

$\frac{5}{4}$

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(8)

$f(x) = 2\sqrt{x}\,(4x^2+3) = mx \quad m = ?$

$x = \sqrt{x} \to 8x^2 + 6 = m\sqrt{x} \longrightarrow 32x^2 = 8x^2 + 6 \to 24x^2 - 6 = 0$

$16x = \frac{m}{2\sqrt{x}} \to m = 32\sqrt{x^3} \qquad x = \frac{1}{2} \leftarrow x^2 = \frac{1}{4} \leftarrow 12x^2 - 3 = 0$

$8 \times \left(\frac{1}{4}\right) + 6 = \frac{m}{\sqrt{2}} \longrightarrow m = 8\sqrt{2} \qquad m = 8\sqrt{2}$

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(9)

$f(x) = \frac{\sqrt{x} - 1}{-2x^2 + x + 1} = mx\sqrt{x} \qquad \frac{0 - (-4x+1)}{(-2x^2+x+1)^2} = \frac{m}{2\sqrt{x}}$

$\frac{-2x^2 + x + 1}{4x - 1} = \frac{m\sqrt{x}}{\frac{m}{2\sqrt{x}}} = 2x \longrightarrow 10x^2 - 2x - 1 = \Rightarrow x^2 - 3x - 10$

$\frac{(x-5)}{10}\frac{(x+2)}{10}$

$\longrightarrow x = -\frac{1}{5}, \frac{1}{2} \longrightarrow x = \frac{1}{2} \longrightarrow y = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}$

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