نوید امینی - تکلیف ۲۴ | ۱۸٫۲۵

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۲- چون شیب مماس ⊕ در $x=a$ برابر $|a|=a$ است:

$A(\alpha, f(\alpha))$

$f(\alpha)=\sqrt{a\alpha-1}=\frac{1}{3}\alpha+\frac{4}{3} \Rightarrow \left(\frac{a}{2}-\frac{4}{3}\right)\times 2=\left(\frac{a}{2}-\frac{4}{3}\right)3=\alpha$

$f'(\alpha)=\frac{a}{2\sqrt{a\alpha-1}}=\frac{1}{3} \to 9a^2=4a\alpha-4 \to a\alpha=\frac{9a^2+4}{4}$

$\frac{9a^2}{2}-9a=\frac{9a^2+4}{4} \to 18a^2-36a=9a^2+4$

$9a^2-36a-4=0$

$a^2-12a-32=0$, $a=\frac{12}{2}=6$

$f(x)=\sqrt{3(x)-1}=4$

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$y=\frac{x^{m+1}+(m+1)x+m-1}{(x+1)^2} \xrightarrow{x=1} \frac{1+m+1+m-1}{4}=\frac{m}{4} \to 2+2m=3m \to m=2$

$m+n=3$

$y=\frac{(x+1)^2}{x+2}\Big|_{x=1} \to \frac{4}{3}=1$ (ق)

$3(1)-2(1)=n \to n=1$

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$f(x)=\frac{(3-\sin x)(\sin^2 x+3\sin x+9)}{(3-\sin x)(2+\sin x)}$

$\left(\frac{x}{3}g-f\right)=\frac{9}{3+\sin x}-\frac{\sin^2 x+3\sin x+9}{3+\sin x}$

$=\frac{-\sin x(\sin x+3)}{3+\sin x}=-\sin x$

$-\cos\left(\frac{5\pi}{3}\right)=-\frac{1}{2}$

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$\left(\log_{\frac{1}{2}}(\sqrt{2})\right)'$

$g(x)=\begin{cases}\frac{1}{x\ln\frac{1}{2}} & x>0 \to -\sqrt{2}x \xrightarrow{x=\sqrt{2}} -2 \\ \text{نمی‌دانم} & \end{cases}$

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۶-

$g(x)=\frac{f(x)-1}{x}$

$\lim_{x\to 0}\frac{f(x)-1}{x}=\frac{0}{0}\xrightarrow{H\circ P}f'(x)=2\left(\frac{-1+\sin x}{1+\sin x}\right)\times\frac{2}{(\sin x+1)^2}\times\cos x$

$2(-1)(2)\times 1=-4$ ✓ (2)

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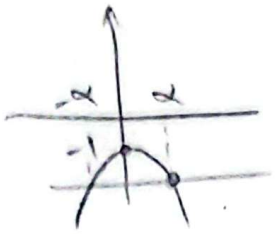
$g=x^2+1\xrightarrow{\text{نسبت به}\ y=x}g_1=x^2-1$



$d_1: g=ax-1$

$y_2=-2x$

$g_1(\alpha)=-2\alpha$

$g_1(-\alpha)=2\alpha$

$-2\alpha\times 2\alpha=-1$

$4\alpha^2=1\to\alpha=\pm\frac{1}{2}$

$\alpha>0\to\alpha=\frac{1}{2}$

$g_1(\frac{1}{2})=-\frac{1}{4}-1=-\frac{5}{4}$

$g=-\frac{5}{4}\to db\to AOB=|\frac{5}{4}|=\frac{5}{4}$ ✓

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$(0,0)\in d\to d: g=ax$

α دلخواه $\to f(\alpha)=a\alpha$

$f'(\alpha)=a$

$2\sqrt{\alpha}(4\alpha^2+3)=a\alpha$

$a=\frac{8\alpha^2+6\alpha}{\sqrt{\alpha}}\xrightarrow{\alpha>0}(8\alpha+6)\sqrt{\alpha}=\frac{11\sqrt{11}}{2}$

$f(x)=\frac{1}{\sqrt{x}}(4x^2+3)+2\sqrt{x}(8\alpha)$ (1,0)

$=\frac{4x^2+3+16x^2}{\sqrt{x}}=\frac{20x^2+3}{\sqrt{x}}$

$2\sqrt{\alpha}(4\alpha^2+3)=\frac{20\alpha^2+3}{\sqrt{\alpha}}\alpha\to 8\alpha^2+6=20\alpha^2+3\to 12\alpha^2=3\to\alpha^2=\frac{1}{4}\to\alpha=\pm\frac{1}{2}$

$\alpha\neq 0$

$\alpha>0\to\alpha=\frac{1}{2}$

$f(x)=\frac{1}{2\sqrt{x}}(-3x^2+x+1)-\sqrt{x}(-6x+1)$

$(-3x^2+x+1)^2$

$=\frac{9x^2-x+1}{2\sqrt{x}(-3x^2+x+1)^2}$

$f(\frac{10\alpha^2+\alpha+1}{\sqrt{\alpha}(-3\alpha^2+\alpha+1)^2})=0\to\frac{\alpha}{-3\alpha^2+\alpha+1}=0$ (2)

$10\alpha^2+\alpha+1=0$ ✗

$f(\alpha)=\frac{\alpha}{2}$ ✓

$10\alpha^2-2\alpha-1=0$

$\alpha^2-2\alpha-10=0\to(\alpha-1)(\alpha+2)\to\alpha=\frac{2}{10}$

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$x = \frac{\sqrt{۵}}{۲} \rightarrow [x] =$

$(f \circ g)'(\frac{\sqrt{۵}}{۲}) = g'(\frac{\sqrt{۵}}{۲}) \times f'(۲^{-}) = ۴\sqrt{۵} \times ۲۴ \times ۴ = ۹۶ \times ۴\sqrt{۵}$ ✱۴۸۴√۵

$g(\frac{\sqrt{۵}}{۲}) = ۲^{+} \rightarrow f(x) = ۸x^{۳} \rightarrow f'(x) = ۲۴x^{۲}$

$g'(x) = \frac{\frac{x}{x\sqrt{x^{۲}-۱}}}{\frac{x^{۲}-۱}{۱}} = \frac{-x}{\sqrt{x^{۲}-۱}(x^{۲}-۱)} = \frac{\frac{\sqrt{۵}}{۲}}{\frac{۱}{۲} \times \frac{۱}{۴}} = ۴\sqrt{۵}$

۵

$$g'(x) \times f'(g(x)) = (fog)'(x)$$

$$x>0 \rightarrow g(x) = \frac{1}{2x^5} \rightarrow f(x) = \frac{-1}{\sqrt[5]{2x}} \rightsquigarrow fog(x) = \frac{-1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}}$$

$$fog(x) = -x \rightarrow fog'(x) = -1 \rightsquigarrow fog'(\sqrt[5]{3}) = 1$$

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$$f(x) = 8x^{\frac{5}{2}} + 4x^{\frac{1}{2}} \rightarrow f'(x) = 20x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$$

معادله‌ی خط مماس در نقطه‌ی $x=a$ برابر است با:

$$y - 2\sqrt{a}(4a^2+3) = \frac{20a^2+3}{\sqrt{a}}(x-a)$$

$$x,y=0 \rightsquigarrow -2\sqrt{a}(4a^2+3) = \frac{20a^2+3}{\sqrt{a}}(-a) \rightsquigarrow 2(4a^2+3) = 20a^2+3$$

$$8a^2+6 = 20a^2+3 \rightarrow 12a^2 = 3 \rightarrow a = \pm\frac{1}{2} \rightsquigarrow a>0 \rightarrow a = \frac{1}{2}$$

$$m = 20\left(2^{-1}\times\frac{3}{2}\right) + 3\left(2^{-1}\left(-\frac{1}{2}\right)\right) = 8\sqrt{2}$$

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$$g(x) = (x^2-1)^{-\frac{1}{2}} \rightarrow g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$$

$$g'\left(\sqrt{\frac{5}{2}}\right) = -\frac{1}{2}(\sqrt{5})\left(\frac{5}{4}-1\right)^{-\frac{3}{2}} \rightarrow -\frac{\sqrt{5}}{2}\left(2^{\frac{-2(-3)}{2}}\right) = -4\sqrt{5}$$

$$g\left(\sqrt{\frac{5}{2}}^-\right) = \frac{1}{\sqrt{\frac{5^-}{4}-1}} = \frac{1}{\sqrt{\frac{1}{4}^-}} = \frac{1}{\frac{1}{2}^-} = 2^+$$

$$f'(2^+) = ((2x)^3)' = 24x^2 = 24\times 4$$

$$fog'\left(\sqrt{\frac{5}{2}}^-\right) = -4\sqrt{5}\times 24\times 4 \xrightarrow{\div -48\sqrt{5}} \frac{2\times 24\times 4 - 4\sqrt{5}}{-48\sqrt{5}} = 1$$