نوید امینی - تکلیف ۲۴

$m = \frac{5-1}{2-0} = \frac{4}{2} = 2 = f'(2)$

۱-

۲- چون شیب مماس ⊕ در $x=a$ مثبت و $a=|a|$

$A(\alpha, f(\alpha))$

$f(\alpha) = \sqrt{a\alpha - 1} = \frac{1}{2}\alpha + \frac{3}{4} \xrightarrow{\times} \left(\frac{a}{2}\right)\left(\frac{a}{2} - \frac{3}{4}\right) = 2 \times \left(\frac{1}{2}\right) \Rightarrow \alpha = 2$

$f'(\alpha) = \frac{a}{2\sqrt{a\alpha - 1}} = \frac{1}{2} \Rightarrow 2a = 2\sqrt{a\alpha - 1} \Rightarrow a\alpha = \frac{3 + a^2}{2}$

$\frac{9a^2}{8} - 9 = \frac{9a^2 + 3}{8} \Rightarrow 9a^2 - 8 = 9a^2 + 3 \Rightarrow 9a^2 - 12a - 3 = 0$

$3a^2 - 4a - 1 = 0, \quad a = \frac{4}{6} = 2$

$f(x) = \sqrt{2(x) - 1} = 3$

۳-

$f = \frac{x^3 + 2(2)x + 2 - 1}{(x+2)^2} \xrightarrow{x=1} \frac{1 - 2 + 4 + 1}{9} = \frac{4}{9} \Rightarrow 4 = m + 2n$

$m + n = 2$

$y = \frac{(x+1)^2}{x+2}\Big|_{x=1} \Rightarrow \frac{4}{3} = 1 \quad (1,1)$

$1 = n + n = (1)^2 - (1) + 3 \Rightarrow n = 1$

۴-

$f(x) = \frac{(2 - \sin x)(\sin^2 x + 2\sin x + 1)}{(2 - \sin x)(2 + \sin x)}$

$(g - f) = \frac{1}{2 + \sin x} - \frac{\sin^2 x + 2\sin x + 1}{2 + \sin x}$

$= \frac{-\sin x (\sin x + 2)}{2 + \sin x} = -\sin x$

$-\cos(\text{مثبت}) = -\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

۵-

$\left(\log_2(\sqrt{x})\right)'$

$\log_2(x) = \begin{cases} -\frac{1}{x\sqrt{\frac{1}{x}}} & x > 0 \\ \text{بقیه} & \end{cases} \rightarrow -\sqrt{2}x \xrightarrow{\text{مشتق}} -\sqrt{2}$

۶-

$$g(x)=\frac{f(x)-1}{x}$$

$$\lim_{x\to 0}\frac{f(x)-1}{x}=\frac{0}{0}\xrightarrow{H\circ P} f'(x)=2\left(\frac{-1+\sin x}{1+\sin x}\right)\times\frac{2}{(\sin x+1)^2}\times\cos x$$

$$2(-1)(2)\times 1=-8$$

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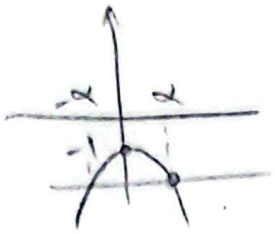
۷-

$g=x^2+1 \xrightarrow[\text{عرض}]{\text{منفی}} y_1=-x^2-1$




$d: y=ax-1$

$y_2=-2x$

$y_1'(\alpha)=-2\alpha$

$y_2'(-\alpha)=2\alpha$

$-2\alpha\times 2\alpha=-1$

$4\alpha^2=1\to\alpha=\pm\frac{1}{2}$

$\alpha>0\to\alpha=\frac{1}{2}$

$y_1'(-\frac{1}{2})=-\frac{1}{2}\cdot 1=-\frac{1}{2}$

$y=-\frac{1}{2}\to d(b)\to |ab|=|1\cdot\frac{1}{2}|=\frac{1}{2}$

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۸-

$(0,0)\in d\to d: y=ax$

$\alpha$ داده‌ای نیست $f(\alpha)=a\alpha$

$f'(\alpha)=a$

$2\sqrt{\alpha}(4\alpha^2+2)=a\alpha$

$a=\frac{8\alpha^2+4\alpha}{\sqrt{\alpha}}\xrightarrow{\alpha=\frac{1}{4}}\frac{(2+\frac{2}{4})}{\frac{1}{2}}=\frac{\frac{11}{2}\sqrt{4}}{4}$

$f(x)=\frac{1}{\sqrt{x}}(4x^2+2)+2\sqrt{x}(8x)$

$=\frac{4x^2+2+16x^2}{\sqrt{x}}=\frac{20x^2+2}{\sqrt{x}}$

$2\sqrt{\alpha}(4\alpha^2+2)=\frac{20\alpha^2+2}{\sqrt{\alpha}}\alpha\to 8\alpha^2+4=20\alpha^2+2\to 12\alpha^2=2\to\alpha^2=\frac{1}{6}\to\alpha=\pm\frac{1}{2}$

$\alpha\neq 0$

$\alpha>0\to\alpha=\frac{1}{2}$

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$f(\sqrt{x}\,f(x))$

$d: y=2x$

$f'(x)=\frac{\frac{1}{2\sqrt{x}}(-12x^2+x+1)-\sqrt{x}(-8x+1)}{(-12x^2+x+1)^2}=\frac{4x^2-x+1}{2\sqrt{x}(-12x^2+x+1)^2}$

$f'\left(\frac{10\alpha^2+\alpha+1}{\sqrt{2\alpha}(-12\alpha^2+\alpha+1)^2}\right)=\frac{2\alpha}{-12\alpha^2+\alpha+1}=16\alpha^2+\alpha+1=-8\alpha^2+2\alpha+2$

$24\alpha^2-\alpha-1=0$

$24\alpha^2-12\alpha+10=0\to(\alpha-1)(\alpha+1)\to\alpha=\frac{1}{24}$

$f(\alpha)=\left(\frac{2}{24}\right)$

$x = \frac{\sqrt{a}}{2} \rightarrow [x] =$

$(f \circ g)'(\frac{\sqrt{a}}{2}) = g'(\frac{\sqrt{a}}{2}) \times f'(2^+) = 4\sqrt{a} \times 2 \times 4 = 32\sqrt{a}$

$g(\frac{\sqrt{a}}{2}) = 2^+ \rightarrow f(x) = ax^{2x} \rightarrow f'(x) = 2ax^{2x}$

$g'(x) = \frac{\frac{x}{x\sqrt{x^2-1}}}{\frac{x^2-1}{1}} = \frac{-x}{\sqrt{x^2-1}(x^2-1)} = \frac{\frac{\sqrt{a}}{2}}{\frac{1}{2} \times \frac{1}{4}} = 4\sqrt{a}$

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