امیر جلالی

جواب ۱)

$$\frac{f(3)-f(1)}{3-1} = \frac{1-\frac{a}{3}-(1-a)}{2} = \boxed{\frac{a}{3}}$$

$$f'(x) = \frac{a}{x^2}$$

$\Rightarrow \frac{a}{3} = \frac{a}{x^2} \Rightarrow x = \pm\sqrt{3}$

→ $-\sqrt{3}$ غیر قابل قبول

→ $+\sqrt{3}$ ✓

جواب ۲)

$y' = 4ax - 5 \xrightarrow{x=A} 4aA - 5 = -1 \Rightarrow aA = 1 \to A = \frac{1}{a}$

$2aA^2 - 5A + 18a = A \Rightarrow aA^2 - 3A + 9a = 0 \to \frac{1}{a} - \frac{3}{a} + 9a = 0$

$1 - 3 + 9a^2 = 0 \Rightarrow 9a^2 = 2 \Rightarrow a = \sqrt{\frac{2}{9}} = \boxed{\frac{\sqrt{2}}{3}}$ ۳

جواب ۳)

$y' = 3x^2 - 12 \Rightarrow x = \pm 2 \Rightarrow$

	-2		$+2$	
$+$		$-$		$+$
↗		↘		↗

→ min نسبی $\Rightarrow x = 2 = 8 - 24 + 2 = -14$

جواب ۴)

$y' = 3x^2 + 2ax - 2b$

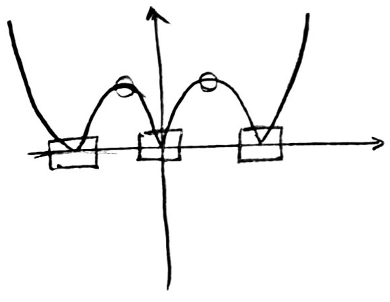
$\xrightarrow{x=0} y' = 0 \Rightarrow b = 0$

$\xrightarrow{x=-2} y' = 0 \Rightarrow 0 = 12 - 4a \Rightarrow \boxed{a = 3}$

$\to y = x^3 + 3x^2 - 4$ → $\left|\begin{matrix}0\\-4\end{matrix}\right.$, $\left|\begin{matrix}-2\\0\end{matrix}\right.$ → فاصله $= \sqrt{(-2)^2 + (-4)^2} \Rightarrow \sqrt{20} = 2\sqrt{5}$

جواب ۵)

$|f(x)| =$



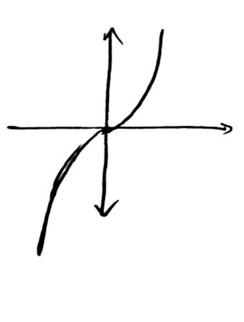
۲○ → max = m

۳□ → min = n

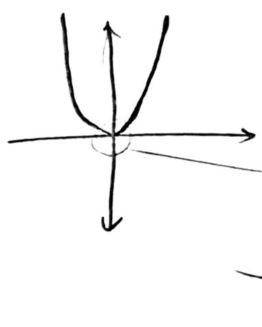
$\Rightarrow \frac{n}{m} = \frac{3}{2}$

جواب ۶)

$f(x) \begin{cases} x \geq 0 \to x^2 + 3x \\ x < 0 \to -x^2 + 3x \end{cases}$ →



$\xrightarrow{|f(x)|}$



→ نقطه بحرانی

۷)

$f'(x)=0 \Rightarrow -(x-a)(x)^{\frac{2}{3}} \Rightarrow x=\dfrac{\frac{2}{3}a+1(0)}{1+\frac{2}{3}}=\boxed{\dfrac{2a}{5}} \leftarrow$ طول max

$\Rightarrow -\left(\dfrac{2a}{5}-a\right)\sqrt[3]{\left(\dfrac{2a}{5}\right)^2}=\dfrac{3a}{5}\sqrt[3]{\left(\dfrac{2a}{5}\right)^2}=\dfrac{3}{2} \Rightarrow \dfrac{a^3}{5^3}\times\dfrac{4a^2}{25}=\dfrac{1}{8} \xrightarrow{\times 5^5} 4a^5=\dfrac{5^5}{8}$

$\Rightarrow a^5=\dfrac{5^5}{2^5} \Rightarrow a=\dfrac{5}{2}=\boxed{2.5}$

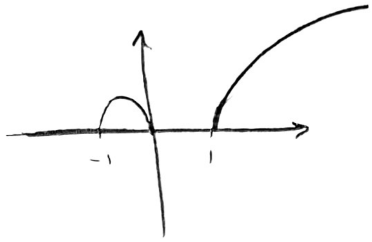
۸)

$x<-1 \rightarrow$ تعریف نشده

$-1\leq x\leq 0 \rightarrow \sqrt{-x^2-x}$

$0<x<1 \rightarrow$ تعریف نشده

$x\geq 1 \rightarrow \sqrt{x^2-x}$

$\longrightarrow$ 

min نسبی $=\boxed{0}$ (n) max نسبی $=\boxed{1}$ (m) بحرانی $=\boxed{1}$ (k) $\Rightarrow \dfrac{1+0}{1-0}=\boxed{1}$

۹)

$y'=\dfrac{m(x-1+m)-(mx+2)}{(\quad)^2}=\dfrac{m^2-m-2}{(x-1+m)^2}\leq 0$

$\Rightarrow m^2-m-2\leq 0 \longrightarrow$ -1, 2: $+\;|\;-\;|\;+$ $\longrightarrow \boxed{m=-1, 0, 1}$

۱۰)

$f(x)\begin{cases} x\geq 0 \xrightarrow{(x\neq 1)} \dfrac{x}{1-x^2} \\ x<0 \longrightarrow \dfrac{x}{1+x^2}\end{cases} \Rightarrow f'(x)=\begin{cases} x\geq 0 \xrightarrow{x\neq 1} \dfrac{1+x^2}{(1-x^2)^2} \\ x<0 \longrightarrow \dfrac{1-x^2}{(1+x^2)^2}\end{cases}$

$\Rightarrow \dfrac{1-x^2}{1+x^2}=0 \Rightarrow 1-x^2=0 \Rightarrow x=\pm 1$ → $+1$ در دامنه نیست
→ -1 ✓

۱ نقطه بحرانی ⟸