

۲. افزین

راد بر حقیقی

$$1) f(x) = 1 - \frac{a}{x} \rightarrow [1, 3] \rightarrow \frac{f(3) - f(1)}{3 - 1} = \frac{-\frac{a}{3} + \frac{a}{1}}{2} = \frac{a}{3}$$

$$f'(x) = \frac{a}{x^2} \rightarrow \frac{a}{x^2} = \frac{a}{3} \rightarrow x^2 = 3 \xrightarrow{x > 1} x = \sqrt{3}$$

$$2) y_p = 2ax^2 - 6x + 11a, \quad y_1 = x \leq 0$$

$$y_2 - y_1 = 2ax^2 - 6x + 11a \rightarrow \Delta = 0 \Rightarrow 6^2 - 4(2a)(11a) = 0$$

$$\rightarrow 144a^2 = 88a \rightarrow a^2 = \frac{1}{2}$$

$$y_2 - y_1 = 2ax^2 - 6x + 11a \rightarrow y_2' - y_1' = 4ax - 6 = 0 \rightarrow a = \frac{6}{4x} \leq 0 \rightarrow a = -\frac{1}{2}$$

$$3) y = x^3 - 12x + 2 \rightarrow y' = 3x^2 - 12 \rightarrow y'' = 6x$$

y''	$-$	0	$+$
y'	$+$	0	$+$
y	\nearrow	\searrow	\nearrow

$$\rightarrow \left. \begin{array}{l} z = 2 \\ y' = 0 \\ y'' > 0 \\ y = -14 \end{array} \right\}$$

تابع پیوسته در $x=2$ مشتق آن در این نقطه از - به + تغییر علامت داده پس بینش میانی است

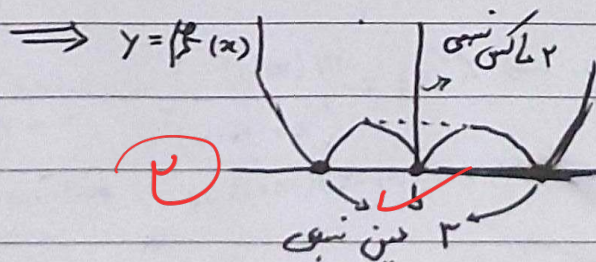
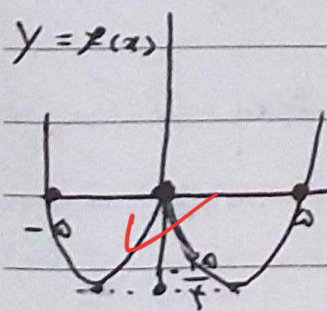
$$4) y = x^3 + ax^2 - 2bx - 4 \rightarrow y' = 3x^2 + 2ax - 2b$$

$$y'(0) = -2b = 0, \quad y'(-2) = 12 - 4a - 2b = 0 \rightarrow a = 3, b = 0$$

$$\rightarrow y = x^3 + 3x^2 - 4 \rightarrow y(0) = -4, \quad y(-2) = 0 \rightarrow d = \sqrt{(0-0)^2 + 4^2} = \sqrt{16} = 4$$

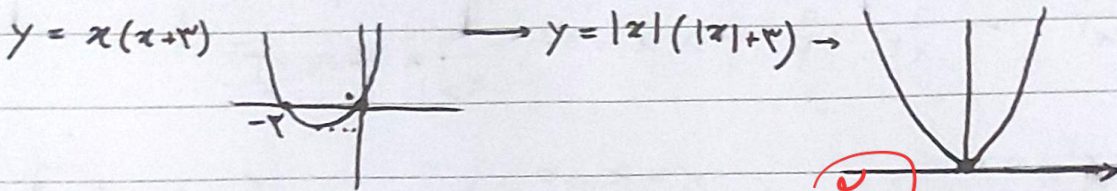
$$d = 2\sqrt{2}$$

$$5) f(x) = x^2 - a|x| \rightarrow y = |f(x)| = \left| |x|^2 - a|x| \right|$$



$$m=2 \rightarrow \frac{n}{m} = \frac{1}{2}$$

8) $f(x) = x(|x|+3) \rightarrow y = |f(x)| = |x(|x|+3)| = |x|(|x|+3)$



$\rightarrow f(x) = x(x+3), x > 0$, $f(x) = x(x-3), x < 0$, $f(0) = 0 \rightarrow f_+(0) = 3, f_-(0) = -3$ قطب افراطی برای دارد

9) $x \in [0, a] \rightarrow f(x) = x^{\frac{1}{p}}(a-x) \rightarrow f'(x) = \frac{1}{p} x^{-\frac{1}{p}}(a-x) - x^{\frac{1}{p}}$

$f'(x) = \frac{1}{p} x^{-\frac{1}{p}}(a - px - px) = 0 \rightarrow x = \frac{a}{p}$

$f(\frac{a}{p}) = (\frac{a}{p})^{\frac{1}{p}} (\frac{a}{p} - \frac{a}{p}) = 0$

$(\frac{a}{p})^{\frac{1}{p}} (\frac{a}{p}) = 1 \rightarrow (\frac{a}{p})^{\frac{p+1}{p}} = 1 \rightarrow a = p$

10) $f(x) = \sqrt{x|x|-x} \rightarrow x(|x|-1) \geq 0 \rightarrow D_f = [-1, 0] \cup [1, +\infty)$

$\rightarrow x > 1: f(x) = \sqrt{x^2-x} \rightarrow f'(x) = \frac{2x-1}{2\sqrt{x^2-x}} = 0 \rightarrow x = \frac{1}{2}$

$\rightarrow -1 \leq x \leq 0: f(x) = \sqrt{-x^2-x} \rightarrow f'(x) = \frac{-2x-1}{2\sqrt{-x^2-x}} = 0 \rightarrow x = -\frac{1}{2}$

عزاسی: $\{(-1, 0), (0, 0), (1, 0), (-\frac{1}{2}, \frac{1}{2})\}$

بین: $\{(-\frac{1}{2}, \frac{1}{2})\}$

کالیس: $\{(-\frac{1}{2}, \frac{1}{2})\}$

$m=1, n=0, k=1 \rightarrow \frac{k m_1 n}{k-n} = m = 1$

9) $f(x) = \frac{mx+2}{x+(m-1)} \rightarrow f'(x) = \frac{m(m-1)-2}{(x+(m-1))^2}$
 $\rightarrow f'(x) < 0 \rightarrow m^2 - m - 2 < 0 \rightarrow (m-2)(m+1) < 0 \rightarrow -1 < m < 2$
مقدار صحیح

10) $f(x) = \frac{x}{1-x|x|} \rightarrow x > 0: f(x) = \frac{x}{1-x^2} \rightarrow f'(x) = \frac{x^2+1}{(1-x^2)^2} > 0$

$\rightarrow x \leq 0 \rightarrow f(x) = \frac{x}{1+x^2} \rightarrow f'(x) = \frac{-x^2+1}{(x^2+1)^2} = 0 \rightarrow x = -1$