Year. Month. Day. Subject.

«بہ نام خدا»

«دکتر کیوان مکارم نسب» تالیف شماره ۲۵

①

$f'(x) = +\frac{a}{x^2} \rightarrow \frac{f(3)-f(1)}{3-1} = \frac{a}{x^2} \rightarrow \frac{(1-\frac{a}{3})-(1-a)}{2} = \frac{a}{x^2}$

$\rightarrow \frac{1-\frac{a}{3}-1+a}{2} = \frac{a}{x^2} \rightarrow \frac{a}{3} = \frac{a}{x^2} \rightarrow x^2 = 3 \rightarrow \boxed{x = \pm\sqrt{3}}$

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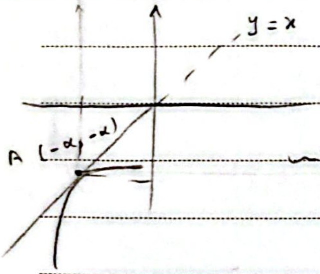
②



① $\begin{cases} f(x) = 2ax^2 - 5x + 18a \\ y = x \end{cases} \rightarrow f(A) = y(A)$, $A(-\alpha, -\alpha)$

② $\begin{cases} f'(x) = 4ax - 5 \\ y' = 1 \end{cases} \rightarrow 4a(-\alpha) - 5 = 1 \rightarrow -4a\alpha = 6 \rightarrow \boxed{a\alpha = -\frac{3}{2}}$ (II)

$2a(-\alpha)^2 - 5(-\alpha) + 18a = -\alpha \rightarrow 2a\alpha^2 + 6\alpha + 18a = 0$ (I)

$\rightarrow 2(a\alpha)\alpha + 6\alpha + 18\left(\frac{-3}{2\alpha}\right) = 0 \rightarrow 2\left(-\frac{3}{2}\right)\alpha + 6\alpha - \frac{27}{\alpha} = 0$

$-3\alpha + 6\alpha - \frac{27}{\alpha} = 0 \rightarrow \frac{27}{\alpha} = 3\alpha \rightarrow \alpha^2 = 9 \xrightarrow[\text{صحیح}]{\alpha > 0} \boxed{-\alpha = -3}$

$\rightarrow a = -\frac{3}{2\times 3} = \boxed{-\frac{1}{2}}$ ✓

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③

$y = x^3 - 12x + 2 \rightarrow y' = 3x^2 - 12 = 3(x^2 - 4) \xrightarrow{y'}$ (-2: + | - | + :2)

x		-2		2	
y'	+		-		+
y	↗		↘	min نسبی	↗

$\rightarrow f(2) = 2^3 - 12(2) + 2 = 8 - 24 + 2 = \boxed{-14}$ ✓

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④

$y = x^3 + ax^2 - 2bx - 4 \xrightarrow{y'} f'(x) = 3x^2 + 2ax - 2b$

$\xrightarrow{\text{ext نسبی}} f'(-2) = 0 \ , \ f(0) = 0 \Longrightarrow f'(-2) = 3(-2)^2 + 2a(-2) - 2b = 0$

$\rightarrow 12 - 4a - 2b = 0 \rightarrow 2a + b = 6$ ①

$f'(0) = 0 \rightarrow -2b = 0 \rightarrow \boxed{b = 0} \xrightarrow{①} \boxed{a = 3} \rightarrow f(x) = x^3 + 3x^2 - 4$ ✓

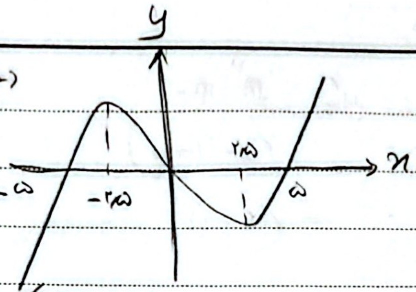
$\rightarrow f(-2) = -8 + 12 - 4 = 0 \rightarrow \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$\rightarrow f(0) = -4 \rightarrow \begin{bmatrix} 0 \\ -4 \end{bmatrix}$

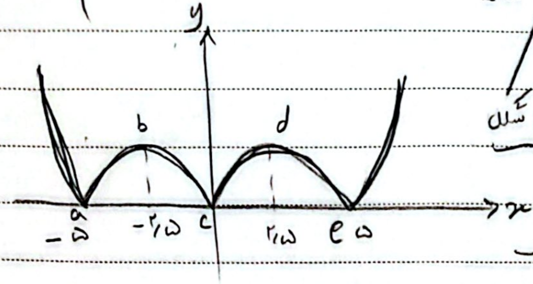
$\xrightarrow{\text{فاصله}} \sqrt{(0+2)^2 + (-4)^2} = \sqrt{16+4} = \boxed{2\sqrt{5}}$

⑤

$$f(x) = \begin{cases} x^2 - 5x & x \geq 0 \\ x^2 + 5x & x < 0 \end{cases} \rightarrow f(x)$$



$|f(x)| \rightarrow$



تعداد Max نسبی: $(m) = b, d \leftarrow$ ۲تا مطابق شکل

تعداد Min نسبی: $(n) = a, c, e \leftarrow$ ۲ تا

$\rightarrow \frac{n}{m} = \frac{2}{2} = ①$ ✓

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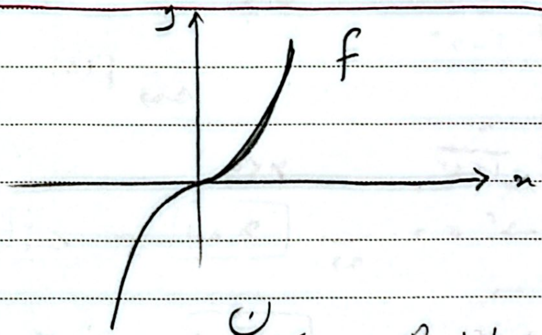
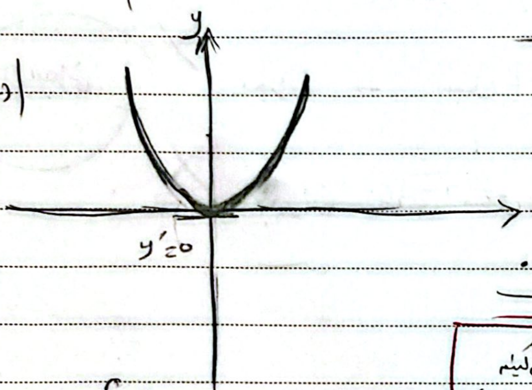
⑥

$$f(x) = \begin{cases} x^2 + 3x & x \geq 0 \\ -x^2 + 3x & x < 0 \end{cases} \rightarrow f$$



$|f(x)| \rightarrow$



این تابع با دامنه $\mathbb{R}$ و پیوسته بودن ← در تمام نقاط، مشتق پذیر است و نقطه بحرانی ندارد.

$$f'(x) = \begin{cases} 2x + 3 & x > 0 \\ -2x + 3 & x < 0 \end{cases}$$

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⑦

$f(x) = \sqrt[3]{x^2}\,(x-a)$ ← فرض می‌کنیم $(x-a) \geq 0$

$\rightarrow f(x) = x^{\frac{2}{3}}(x-a) = x^{\frac{5}{3}} - ax^{\frac{2}{3}} \Rightarrow f'(x) =$

$\frac{5}{3}x^{\frac{2}{3}} - \frac{2}{3}ax^{-\frac{1}{3}} = \frac{1}{3}x^{-\frac{1}{3}}(5 - 2ax) = 0 \rightarrow 5 - 2ax = 0 \rightarrow 2ax = 5 \rightarrow x = \frac{5}{2a}$

$F(\frac{5}{2a}) = \frac{3}{2} \rightarrow \sqrt[3]{\frac{25}{4a^2}}\left(\frac{5}{2a} - \frac{2a^2}{2a}\right) = \frac{3}{2}$

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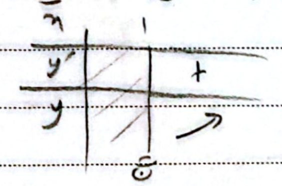
⑧

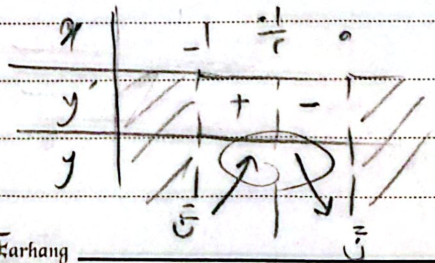
$$f(x) = \begin{cases} \sqrt{x^2 - x} & x \geq 1 \\ \sqrt{-x^2 - x} & -1 \leq x \leq 0 \end{cases}$$

طبق دامنه زیر رادیکال داریم:



$\rightarrow \frac{4(1) + 0}{4 - 0} = \frac{4}{4} = ①$ ✓

$$\rightarrow f'(x) = \begin{cases} \frac{2x - 1}{2\sqrt{x^2 - x}} & x > 1 \\ \frac{-2x - 1}{2\sqrt{-x^2 - x}} & -1 < x < 0 \end{cases}$$



بحرانی $\Leftarrow x = -\frac{1}{2}$ و ۰ و ۱- و ۱

$\rightarrow k = 4$

$x = -\frac{1}{2} \rightarrow$ Max نسبی $\rightarrow m = 1$

$n = 0 \rightarrow$ Min نسبی ندارد

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(۹)

$y = \frac{mx+2}{x-1+m} \rightarrow y' = \frac{m^2-m-2}{(x-(1-m))^2} = \frac{(m+1)(m-2)}{(x-(1-m))^2}$ ① $\rightarrow x=1 \rightarrow y'=0$

$(1-(1-m))^2 = 0 \rightarrow \boxed{m=0}$

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(۱۰)

$f(x) = \begin{cases} \frac{x}{1-x^2} & x \geq 0 \\ \frac{x}{1+x^2} & x < 0 \end{cases} \rightarrow f'(x) = \begin{cases} \frac{x^2+1}{(1-x^2)^2} & x > 0 \\ \frac{1-x^2}{(1+x^2)^2} & x < 0 \end{cases}$

① $\rightarrow 1-x^2 = 0 \xrightarrow{x>0} \boxed{x=1} \rightarrow$ مشتق ناپذیر $\rightarrow$ بحرانی ✓

② $\rightarrow 1-x^2 = 0 \xrightarrow{x<0} \boxed{x=-1}$ مشتق برابر صفر $\rightarrow$ بحرانی ✓

(۲ تا بحرانی)

③ $\rightarrow 1+x^2 = 0 \rightarrow$ همواره مثبت X