لوبیا فلفلیان

به نام خدا

۱۹٫۷۵ آفرین

DATE / / SUBJECT:

① $f(1) = 1 - a \,,\, f(3) = 1 - \frac{a}{3} \,,\, f(x) = ax^{-2} \to \frac{\frac{2}{3}a}{2} = \frac{a}{3} = ax^{-2} \to \frac{1}{x^2} = \frac{1}{3} \xrightarrow{x \in [1,3]}$ ۲

$\boxed{x = \sqrt{3}}$

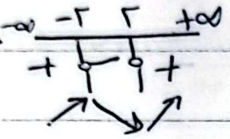
②

$2ax^2 - 5x + 18a = x \to 2ax^2 - 6x + 18a = 0 \to ax^2 - 3x + 9a = 0 \to \Delta = 0 \to$

$9 - 36a^2 = 0 \to a^2 = \frac{1}{4} \to a = \pm\frac{1}{2} \qquad 4ax - 5 = 1 \to x = \frac{6}{4a} < 0 \to a < 0 \to$

$\boxed{a = \frac{-1}{2}}$ ۲

③

$y' = 3x^2 - 12 = 0 \to x = \pm 2 \to$  $\to x = 2 \to y = 8 - 24 + 2 = \boxed{-14}$ ۲

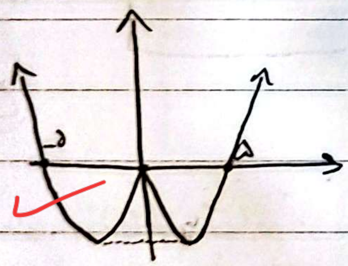
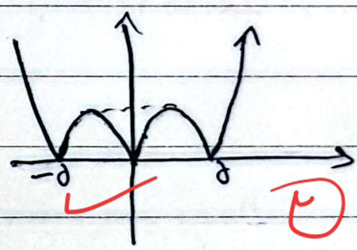
④

$y' = 3x^2 + 2ax - 2b \to x = 0 \to y' = 0 \to b = 0$

$\to x = -2 \to y' = 0 \to 12 - 4a = 0 \to a = 3$ $\Big\} \Rightarrow y = x^3 + 3x^2 - 4$

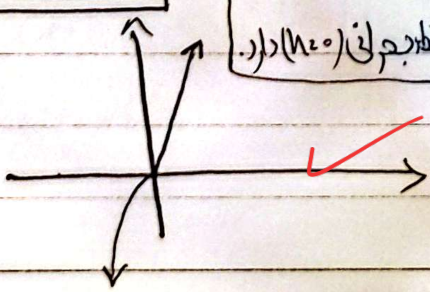
$\to A\begin{vmatrix}0\\-4\end{vmatrix} , B\begin{vmatrix}-2\\0\end{vmatrix} \to d_{AB} = \sqrt{2^2 + 4^2} = \boxed{2\sqrt{5}}$ ۲

⑤

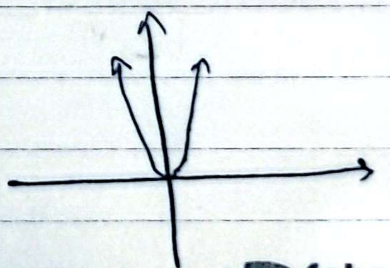
$y = x^2 - 5|x|$  $\to$ $y = |f(x)|$  $\to$ ۲

$m = 2 , n = 3 \to \boxed{\frac{n}{m} = \frac{3}{2}}$

⑥

$f(x) = x(|x| + 2)$ 

یک نقطه بحرانی $(x=0)$ دارد.

$y = |f(x)|$  ۲

اولیا فاطمیان

DATE / / SUBJECT:

(۷) [crossed-out working]

$0 \le x \le a \to f(x) = \sqrt{x^3}(-x+a) \to f'(x) = \frac{3}{2}x^{\frac{1}{2}}(-x+a) + (-1)(x^{\frac{3}{2}}) = x^{\frac{1}{2}}(-\frac{3}{2}x + \frac{3}{2}a - x)$

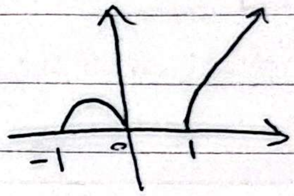
$= x^{\frac{1}{2}}(-\frac{5}{2}x + \frac{3}{2}a) \to$ (sign table: 0, $\frac{3}{5}a$; $- \; + \; -$) $\to f(\frac{3}{5}a) = 1.5 \to (\frac{3}{5}a)^{\frac{3}{2}} \times (\frac{2}{5}a) = \frac{3}{2} \to$

$a \times (\frac{3}{5}a)^{\frac{3}{2}} = \frac{5}{2} \to a^{\frac{5}{2}} \times (\frac{4}{25})^{\frac{1}{2}} = \frac{5}{2} \to a^{\frac{5}{2}} = (\frac{5}{2})^{\frac{5}{2}} \to$ **$a = 2.5$**

(۸)

$0 \le x \quad f(x) = \sqrt{x^2 - x}$

$0 > x \quad f(x) = \sqrt{-x^2 - x}$



$m = 1, n = 0, k = 3 \to \frac{km+n}{k-n} = \frac{3}{3} = 1$

(۹)

$y = \frac{mx+2}{x+m-1} \to y' = \frac{m(m-1)-2}{(x+m-1)^2} = \frac{(m-2)(m+1)}{(x+m-1)^2} \to$ (-1, 2: $+ \; - \; +$)

$m \in [-1, 2) \to 1-m \ne 1 \to 0 \ne m \to$ **$m \in [0, 2)$** $\to m = 0, 1 \to$

به ازای دو مقدار صحیح

(۱۰)

$0 \le x \quad \frac{x}{1-x^2} \to 1 - x^2 = 0 \to$ **$x = 1$** بحرانی , $\frac{1-x^2-(-2x)(x)}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2} \ne 0$

$0 > x \quad \frac{x}{1+x^2} \to 1 + x^2 \ne 0 \to \frac{1+x^2-(2x^2)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} \to$ $x = -1$ بحرانی

دو نقطه بحرانی دارد $(x = \pm 1)$ ← $x = 1$ نداریم

یک نقطه ی بحرانی دارد $(x = -1)$