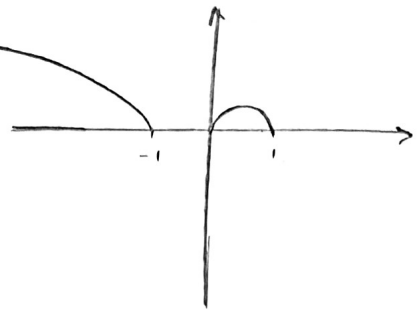
امیر جلالی

۱۸٫۵

(۱)

$\sqrt{x(1-|x|)} \longrightarrow$



$$\begin{cases} \sqrt{-x^2+x} & x \geq 0 \\ \sqrt{x^2+x} & x < 0 \end{cases}$$

$\min$ نسبی $= 0 \to n$

$\max$ نسبی $= 1 \to m$

نقاط برانی $= 2 \to k$

$\Rightarrow m+n+k = 5$ ۲

۲

(۲)

$f(x) = \text{اکیدا صعودی} + \text{اکیدا صعودی} = \text{اکیدا صعودی} \Rightarrow \begin{cases} \min \to \text{ابتدا بازه} \ (\text{min}) \\ \max \to \text{انتها بازه} \ (\text{min}) \end{cases}$

۱۵

$D_f = [0, \frac{a}{2}]$

$\Rightarrow \min \times \max = \sqrt{a} \times \sqrt{\frac{a}{2}} = \sqrt{\frac{a^2}{2}} = \sqrt{12} \Rightarrow a^2 = 48 \Rightarrow a = \pm\sqrt{48}$

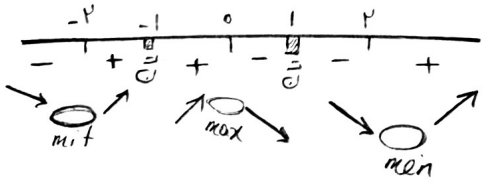
منفی غیر قابل قبول

$\Rightarrow a = +\sqrt{48} \Rightarrow [a] = 6$ ۲

(۳)

$$f(x) \begin{cases} x \geq 2, x < -2 \to \frac{x^2(x^2-4)}{x^2-1} \\ -2 \leq x \leq 2 \to -\frac{x^2(x^2-4)}{x^2-1} \end{cases} \longrightarrow y' = \frac{2x^5-8x^3+8x}{(x^2-1)^2} = 0 \Rightarrow 2x(\underbrace{x^4-2x^2+4}_{\Delta<0}) = 0$$

۳



$\longrightarrow$ مطابق شکل ۲

(۴)

$f(0) = 0 \Rightarrow d = 0$

$f'(0) = 0 \Rightarrow c = 0$

$\Rightarrow f(x) = ax^3 + bx^2 \Rightarrow \begin{cases} f(1) = a+b = 1 \\ f'(1) = 3a+2b = 0 \end{cases} \Rightarrow \begin{cases} a = -2 \\ b = 3 \end{cases} \Rightarrow ab = -6$ ۲

(۵)

$f(x) = -x^3+3x \longrightarrow f'(x) = -3x^2+3 = 0 \Rightarrow \begin{cases} x = 1 \\ x = -1 \end{cases}$

$f(1) = 2$

$f(-1) = -2 \to \min$ ۲

$f(\sqrt{2}) = 0$

$f(-1,a) = -\frac{9}{8}$

$y = -x^3 + 3ax^2 + b \longrightarrow y' = -3x^2 + 6ax \xrightarrow{x=-1} -3 - 6a = 0 \Rightarrow a = -\frac{1}{2}$ ۴

$y(-1) = 1 \Rightarrow 1 - \frac{3}{2} + b = 1 \Rightarrow b = +\frac{3}{2} \Longrightarrow \frac{b}{a} = \frac{+\frac{3}{2}}{-\frac{1}{2}} = -3$ ۲

$min \longrightarrow -\frac{b}{2a} = x = -\frac{1}{3}$ ۷

مجانب افقی $= y = \frac{-a+1}{a+1} \Longrightarrow \frac{a}{a+1} = -\frac{1}{3} \Rightarrow 3a = -a - 1 \Rightarrow a = -\frac{1}{4}$

مجانب قائم $= x = -\frac{a}{a+1} \Rightarrow -\frac{1}{4}x + 3 = 0 \Rightarrow x = 12$ ۱٫۷۵

مجانب افقی $= y = 3 \longrightarrow \lim_{x \to \infty} \frac{bx^2+7}{4x^2+ax+1} = 3 \Rightarrow \frac{b}{4} = 3 \Rightarrow b = 12$

مجانب قائم $= x = -\frac{1}{2} \longrightarrow \lim_{x \to -\frac{1}{2}} \frac{bx^2+7}{4x^2+ax+1} = \infty \Rightarrow 4\left(\frac{1}{4}\right) + -\frac{a}{2} + 1 = 0 \Rightarrow a = 4$ ۲

$\frac{b}{a} = \frac{12}{4} = 3$ ۱

$f'(x) = \frac{4x^3(x^3-8) - 3x^2(x^4)}{(x^3-8)^2} \Rightarrow 4x^6 - 32x^3 - 3x^6 = 0 \longrightarrow x^6 - 32x^3 = 0$ ۹

$x^3(x^3 - 32) = 0$

$x = 0, \quad x^3 = 32 \Rightarrow x = \sqrt[3]{32} = 2\sqrt[3]{4}$

$f'(x)$: $+$ on $(-\infty, 0)$, $-$ on $(0, 2)$, $-$ on $(2, 2\sqrt[3]{4})$, $+$ on $(2\sqrt[3]{4}, +\infty)$

صعودی — نزولی — نزولی — صعودی

$\Longrightarrow [0, 2) \cup (2, 2\sqrt[3]{4}]$ ۱٫۵

$\min$ طول بازه $= |2\sqrt[3]{4} - 2| = 2(\sqrt[3]{4} - 1)$

۲

$f(x) = \sqrt{x} + \sqrt{a-2x} \rightarrow Df \quad 0 \le x \le \frac{a}{2}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{2\sqrt{a-2x}} \quad f'=0 \rightsquigarrow \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{a-2x}} \rightarrow 4x = a-2x \rightsquigarrow x = \frac{a}{6}$

$x=0 \rightarrow f(0) = \sqrt{a}$

$x = \frac{a}{2} \rightarrow f(\frac{a}{2}) = \frac{\sqrt{a}}{\sqrt{2}}$ نقطه min

$x = \frac{a}{6} \rightarrow f(\frac{a}{6}) = \sqrt{\frac{a}{6}} + \sqrt{\frac{4a}{6}} = \frac{3\sqrt{a}}{\sqrt{6}}$ نقطه max

$\min \times \max = 3$

$\frac{3 \times a}{\sqrt{12}} = \sqrt{12}$

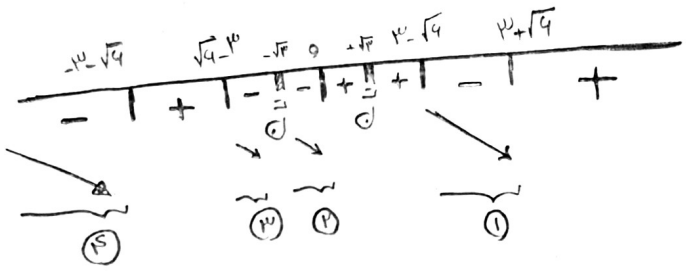
$\hookrightarrow \boxed{a = 4}$

$f'(x) = \frac{4x^3(x^2-3) - 2x(x^4-3)}{(x^2-3)^2} \Rightarrow 4x^5 - 12x^3 - 2x^5 + 6x = 0 \Rightarrow 2x^5 - 12x^3 + 6x = 0$ | ۱۵

$x(x^4 - 6x^2 + 3) = 0$

$x = 0$ ، $x^2 = 3 \pm \sqrt{6}$

$\pm(3-\sqrt{6})$ ، $\pm(3+\sqrt{6})$



۱۵

۴ بازه، ابتدا نزولی

۷

$x_{min} = -\frac{b}{2a} = \frac{-1}{2(\frac{3}{2})} = -\frac{1}{3}$

شیب مماس = $-\frac{d}{c} = \frac{1-a}{1+a} = -\frac{1}{3} \rightarrow 3-3a = -1-a \rightarrow 2a = 4 \rightarrow a = 2$

$y = \frac{2x+3}{3x+1} \rightarrow y=0 \rightarrow x = -\frac{3}{2}$

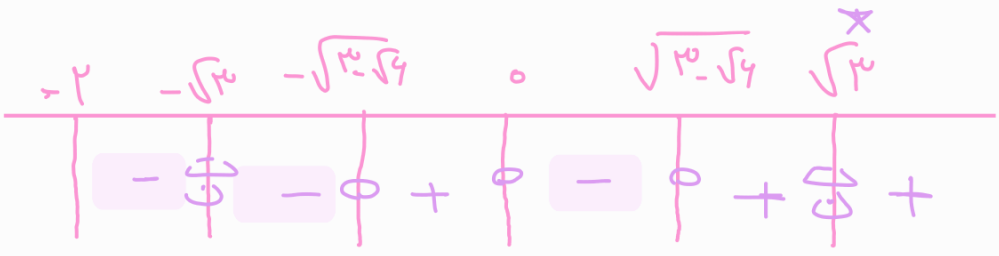
$$f'(x) = \frac{4x^3(x^2-3) - 2x(x^4-3)}{(x^2-3)^2} = \frac{2x\left[(2x^4-6x^2) - (x^4-3)\right]}{(x^2-3)^2}$$

$$2x^5 - 12x^3 + 6x = 0 \longrightarrow 2x(x^4 - 6x^2 + 3) = 0 \quad \hookrightarrow x = 0$$

$$\hookrightarrow x^2 = t$$

$$t^2 - 6t + 3 = 0 \longrightarrow t = \frac{6 \pm \sqrt{24}}{2} \longrightarrow x = \pm\sqrt{3-\sqrt{6}}$$

$-2 < x < 2$



در ۳ بازه اکیداً نزولی است!