

۱. اثبات های نوشتاری!

$$f(x) = \cos^3(2x) + ax^2 + b \rightarrow f'(x) = -3\cos^2(2x) \cdot \sin(2x) \cdot 2 + 2ax$$

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = 0 \rightarrow \lim_{x \rightarrow 0^+} \frac{\cos^3(2x) + \sqrt{x^2} + b}{x} = 0 \rightarrow \lim_{x \rightarrow 0^+} \cos^3(2x) + \sqrt{x^2} + b = 0 \rightarrow 1 + b = 0 \rightarrow b = -1$$

$$\lim_{x \rightarrow 0^-} \frac{f'(x)}{x} = 2 \rightarrow \lim_{x \rightarrow 0^-} \frac{-3\cos^2(2x) \cdot \sin(2x) \cdot 2 + 2ax}{x} = 2$$

$\lim_{x \rightarrow 0^-} 2\sqrt{x^2} - 12 + 2a = 2 \rightarrow a = 5$

$y = x^2 - 1$ \rightarrow وقتی خط مماسی خود را از این نمودار با قطع کرد آن دو نقطه طول یکی متفاوت و عرض یکی مساوی دارند

$y' = 2x \rightarrow \left| \frac{m - m'}{1 + mm'} \right| = \tan \alpha$
 $-m = 0 \cdot m'$

$\left| \frac{2m}{1 - m^2} \right| = \tan \alpha$
 $1 - m^2 = 0 \rightarrow m = 1$

$x = \frac{1}{2} \rightarrow y = -\frac{3}{4}$
 $x = -\frac{1}{2} \rightarrow y = -\frac{3}{4}$
 $-\frac{3}{4} - \frac{3}{4} = -\frac{3}{2}$

$x' = -\frac{1}{2} \quad x = \frac{1}{2} \leftarrow m = 2x \leftarrow m = 1$

$(0, 0) \rightarrow (-1, 2) \rightarrow (2, 0) \rightarrow (4, 4)$

$m = \frac{4 - (-12)}{2 - (-0.5)} = 9$

$f(x) = \frac{-3}{2x-1} \rightarrow f(0) = \frac{-3}{-1} = 3$

$9x - 9 = \frac{a}{2x-1}$
 $12x^2 - 22x + 9 - a = 0$
 $22x - 22 = 0 \rightarrow x = 1$
 $-3 - a = 0 \rightarrow a = -3$

$y = 2x + b$
 $y = \frac{x+a}{ax+1}$

$2ax^2 + (ab+1)x + b - a = 0 \xrightarrow{x=1} a + ab + b = -1$

$2ax^2 + (ab+1)x + b - a = 0 \xrightarrow{x=1} a + ab + b = -1$

$2ax^2 + (ab+1)x + b - a = 0 \xrightarrow{x=1} a + ab + b = -1$

$f(x) = \sin x + \frac{1}{x} \cos x$
 $g(x) = \frac{1}{x} \sin x$

$f'(x) = \cos x - \frac{1}{x^2} \sin x$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} (x - \frac{\pi}{4}) \xrightarrow{y=0} -\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} (x - \frac{\pi}{4}) \rightarrow -1 = x - \frac{\pi}{4}$

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \rightarrow f'(x) = 6x^2 - 6x - 12 = 0$$

$\rightarrow x=2 \rightarrow f(2) = -9$
 $\rightarrow x=-1 \rightarrow f(-1) = 9$

$$6x^2 - 6x - 12 = -9 \rightarrow 6x^2 - 6x - 3 = 0$$

$$\Delta \geq 0 \rightarrow \text{نقطه } \textcircled{2}$$

$$m = \frac{\Delta - (-12)}{-1 - 2} = 9$$

$$y = kx^3 + (k+1)x^2 \rightarrow x^2(kx + k+1) > 0 \rightarrow \frac{2k+2}{2} > 0$$

$$y' = 3kx^2 + 2(k+1)x$$

$$y'' = 6kx + 2k+2 = 0 \rightarrow x = \frac{-2k-2}{6k} = \frac{-k-1}{3k} < 0$$

خط مماس
 در نقطه
 (-1, 9)
 از منحنی عبور
 می کند پس این
 نقطه نقطه
 عطف تابع است

$\frac{a}{b} = \frac{3}{5}$

$$x = x^3 + dx^2 + bx - 1 \rightarrow y' = 3x^2 + 2dx + b \rightarrow y'' = 6x + 2d = 0$$

$$x = -1 \rightarrow -6 + 2d = 0 \rightarrow d = 3$$

$$f(-1) = -1 \Rightarrow x^3 + 3x^2 + bx - 1 \xrightarrow{x=-1} -1 + 3 - b - 1 = -1$$

$$b = 0$$

$$\frac{a}{b} = \frac{3}{0}$$

$$f(x) = x^3 + ax^2 + bx + c \xrightarrow{x=0} c = f$$

$$f'(x) = 3x^2 + 2ax + b = 0$$

$x=0 \rightarrow b=0$
 $x = -\frac{2a}{3}$

$$f\left(-\frac{2a}{3}\right) = 0 = \frac{-1 \cdot 16a^3}{27} + \frac{4a^3}{9} + f = 0$$

$$\frac{4a^3}{9} = \frac{16a^3}{27} \rightarrow a^3 = 27 \rightarrow a = 3$$

$$f(x) = x^3 - 6x^2 + 0 \rightarrow f'(x) = 3x^2 - 12x \rightarrow f''(x) = 12x - 12 = 0$$

$x=1 \rightarrow f(1) = 0$
 $x=-1 \rightarrow f(-1) = 0$

$m_{CD} = 0$

$D(1, 0)$
 $C(-1, 0)$

$m_{AB} = 0$

$A(-\sqrt{3}, -f)$
 $B(\sqrt{3}, -f)$

دو خط موازی بود، بنابراین
 با هم موازی هستند