«به نام خدا»

دکتر کیوان (۸) مکانیک شب ۲۲ — تکلیف شماره ۲۷

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$\lim\limits_{x\to 0^+} \dfrac{\cos^3(2x)+ax^2+b}{x} = 0$ $\frac{0}{0}$ صورت $=0$ → $1+0+b=0$ → $\boxed{b=-1}$

→ $f'(x) = 3\cos^2(2x)\times(-2\sin 2x) + 2ax$ → $\lim\limits_{x\to 0} \dfrac{-3\cos 2x\cdot \sin 4x + 2ax}{x} = 2$

$\xrightarrow[HOP]{\frac{0}{0}}$ $\lim\limits_{x\to 0} \dfrac{(6\sin 2x\cdot \sin 4x)+(4\cos 4x\cdot(-3)\cos 2x)+2a}{1} = 2$

→ $0+(4\times -3)+2a = 2$ → $2a-12=2$ → $\boxed{a=7}$ → $a+b=6$

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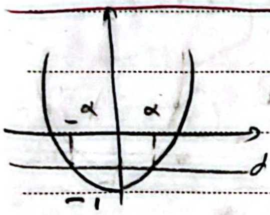

$A\begin{bmatrix}\alpha\\ \alpha^2-1\end{bmatrix}$, $B\begin{bmatrix}-\alpha\\ \alpha^2-1\end{bmatrix}$ → $m_A = 2\alpha$, $m_B = -2\alpha$

→ $m_A\cdot m_B = -1$ → $-4\alpha^2=-1$ → $\alpha = \frac{1}{2}$

$\alpha^2-1 = \frac{1}{4}-1 = -\frac{3}{4}$ → $-\frac{3}{4}-\frac{3}{4} = -\frac{3}{2} = -1.5$ مجموع عرض ها

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$f(x) = \dfrac{a}{2x-1}$ → $f'(x) = \dfrac{-a(2)}{(2x-1)^2} = \dfrac{-2a}{(2x-1)^2} = 6$

$\begin{bmatrix}2.5\\6\end{bmatrix}$, $\begin{bmatrix}-0.5\\-12\end{bmatrix}$ → شیب $= \dfrac{6-(-12)}{2.5-(-0.5)} = \dfrac{18}{3} = 6$

$a = -3(2x-1)^2$ → $f(x) = \dfrac{-3(2x-1)^2}{2x-1} = -3(2x-1)$

$f(5) = -3(\cancel{10}-1) = -27$

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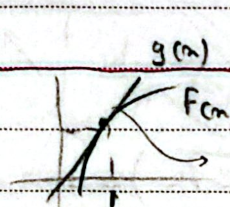

$f(x) = \dfrac{x+a}{ax+1}$, $g(x) = 2x+b$ → $f'(x) = \dfrac{1-a^2}{(ax+1)^2} = 2$ $\xrightarrow{x=1}$

$\dfrac{1-a^2}{(a+1)^2} = 2$ → $1-a^2 = 2(a^2+2a+1)$

→ $1-a^2 = 2a^2+4a+2$ → $3a^2+4a+1=0$ → $\boxed{a=-\frac{1}{3}}$, $a=-1$ ✗ تابع ثابت

$g(1)=f(1)$ → $2+b = \dfrac{1-\frac{1}{3}}{-\frac{1}{3}+1}$ ⇒ $2+b = \dfrac{\frac{2}{3}}{-\frac{2}{3}}$ ⇒ $2+b=-1$ → $\boxed{b=-3}$

→ $a-b = -\frac{1}{3}+3 = \frac{8}{3}$

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$g(x)=f(x) \Rightarrow \frac{3}{2}\sin x = \sin x + \frac{1}{2}\cos x \Rightarrow -\frac{1}{2}\sin x + \frac{1}{2}\cos x = 0$

$\sin x = \cos x \Rightarrow \boxed{x = \frac{\pi}{4}} \Rightarrow f'(x) = \cos x - \frac{1}{2}\sin x \rightarrow \begin{bmatrix} \frac{\pi}{4} \\ \frac{3\sqrt{2}}{4} \end{bmatrix}$

طول نقطه برخورد

$f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{1}{2}(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$ شیب خط مماس

$(y - \frac{3\sqrt{2}}{4}) = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) \Rightarrow y = \frac{\sqrt{2}}{4}x - \frac{\sqrt{2}\pi}{16} + \frac{3\sqrt{2}}{4} \xrightarrow{y=0}$

$x = \dfrac{\frac{\pi\sqrt{2}}{16} - \frac{3\sqrt{2}}{4}}{\frac{\sqrt{2}}{4}} = \dfrac{\cancel{\frac{\sqrt{2}}{4}}(\frac{\pi}{4} - 3)}{\cancel{\frac{\sqrt{2}}{4}}} \Rightarrow \boxed{x = \frac{\pi}{4} - 3}$

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$f(x) = 2x^3 - 3x^2 - 12x + 1 \Rightarrow f'(x) = 6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

x		-1		$+2$	
y'	$+$	0	$-$	0	$+$
y	$\nearrow$		$\searrow$		$\nearrow$

max نسبی   min نسبی

$A = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -19 \end{bmatrix}$

$\Rightarrow m_{AB} = \frac{8-(-19)}{-1-2} = \frac{27}{-3} = -9$

$\Rightarrow f'(x) = -9 \Rightarrow 6x^2 - 6x - 12 = -9$

$\Rightarrow 6x^2 - 6x - 3 = 0 \Rightarrow \Delta > 0 \Rightarrow$ ۲ ریشه دارد ⇐ ۲ نقطه روی منحنی با این شرط وجود دارد

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نقطه‌ی عطف درناحیه سوم $= x_{\text{عطف}} < 0 , \; y_{\text{عطف}} > 0 \Rightarrow x_{\text{عطف}} = -\frac{b}{3a} = \frac{-(k+1)}{3k} < 0$ ①

$\Rightarrow$ -1 0 : $-$ | $+$ | $-$ $\Rightarrow (-\infty, -1) \cup (0, +\infty)$

$f(x) = x^2(kx + k+1) \Rightarrow f(x_{\text{عطف}}) = \left(\frac{-(k+1)}{3k}\right)^2 \left(-k\left(\frac{k+1}{3k}\right) + k+1\right)$

② $\Rightarrow f(x_{\text{عطف}}) = \frac{(k+1)^3}{9k^2} \times \frac{2}{3} > 0 \Rightarrow$ -1 0 : $-$ | $+$ | $+$ $\Rightarrow (-1, +\infty)$

$k \in (0, +\infty) \Rightarrow$ به ازای هیچ مقدار k منفی صحیح نقطه عطف در ناحیه دوم مختصات نیست.

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$\begin{bmatrix}-1\\-4\end{bmatrix}$ = نقطه عطف $\Rightarrow$ ① $f(-1)=-4$ و ② $f''(-1)=0$

$f(x)=x^3+ax^2+bx-1 \Rightarrow f'(x)=3x^2+2ax+b \Rightarrow f''(x)=6x+2a$

$f''(-1)=0 \Rightarrow -6+2a=0 \to \boxed{a=3}$ ✓

$f(-1)=-1+a-b-1=-4 \xrightarrow{a=3} -1+3-b-1=-4 \to \boxed{b=5}$ ✓

$\Rightarrow \frac{a}{b}=\frac{3}{5}=0.6$ ✓

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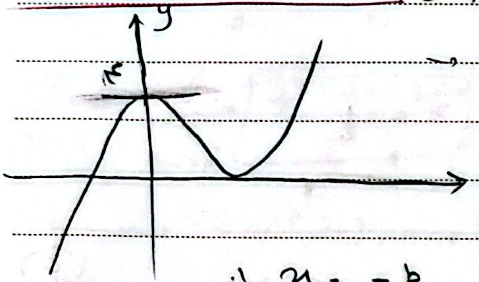
① $f(0)=4$ و ② $f'(0)=0$



$\Rightarrow f(0)=c \Rightarrow \boxed{c=4}$ ✓ , $f'(x)=3x^2+2ax+b$

$f'(0)=b=0 \to \boxed{b=0}$ ✓

$\frac{y_{max}+y_{min}}{2}=y_{\text{عطف}}=\frac{4+0}{2}=2$

$x_{\text{عطف}}=-\frac{b}{3a} \Rightarrow f(x)=x^3+ax^2+4 \Rightarrow x_{\text{عطف}}=\frac{-a}{3}$

$\Rightarrow f(-\frac{a}{3})=2 \to \frac{-a^3}{27}+a(\frac{a^2}{9})+4=2 \to \frac{-a^3}{27}+\frac{3a^3}{27}+4=2$

$\frac{2a^3}{27}=-2 \Rightarrow a^3=-27 \to \boxed{a=-3}$ ✓ $\Rightarrow f(x)=x^3-3x^2+4$

$f'(x)=3x^2-6x=0 \Rightarrow 3(x)(x-2)=0 \Rightarrow \boxed{x_{min}=2}$ ✓

← طبق شکل مشخص است که x=2 نقطه مینیمم نسبی است.

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$f(x)=x^4-6x^2+5 \Rightarrow f'(x)=4x^3-12x=0 \to 4x(x^2-3)$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
y'	$-$	0	$+$	0	$-$	0	$+$
y	↘		↗		↘		↗
		A				B	

$A\begin{bmatrix}\sqrt{3}\\-4\end{bmatrix}$ ✓ , $B=\begin{bmatrix}-\sqrt{3}\\-4\end{bmatrix}$ ✓ $\to m_{AB}=0$ ①

$f''(x)=12x^2-12=12(x-1)(x+1)=0$

x		-1		1	
y''	$+$	0	$-$	0	$+$
y	∪		∩		∪
		عطف		عطف	

$\Rightarrow D=\begin{bmatrix}-1\\0\end{bmatrix}$, $C=\begin{bmatrix}1\\0\end{bmatrix} \to m_{CD}=0$ ②

①,② $\tan\alpha=\left|\frac{m-m'}{1+mm'}\right|=0$ ✓ $\to \boxed{\alpha=0^\circ}$

صفر درجه است.

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