امیرهاشمی

Date:

(1)

$\lim_{x\to 0} \frac{f(x)}{x} = 0 \to f(0) = 0 \to f(0) = 1 + a + b = 0 \Rightarrow b = -1$

$f'(x) = 3\cos^2(2x)(-2\sin 2x) + 2a = -6\underbrace{(1-\sin^2 2x)(\sin 2x)}_{\sin 2x - \sin^3 2x} + 2ax$

$= 6\sin^3 2x - 6\sin 2x + 2a$

$f''(x) = 18\sin^2 2x \times 2\cos 2x - 12\cos 2x + 2a$

$\lim_{x\to 0} f''(x) = -12 + 2a = -2 \to \boxed{a = 5} \to a + b = 4$

(2)

شیب خط مماس اول $m_1 = f'(x_A) = 2x_A$
شیب خط مماس دوم $m_2 = f'(x_B) = 2x_B$ $\Big\}$ $m_1 m_2 = -1$

$2x_A \, 2x_B = -1 \Rightarrow x_A x_B = \frac{-1}{4}$

و می‌دانیم چون نقطه $x = 0$ محور تقارن سهمی است $\Leftarrow x_A = -x_B$

$x_A(-x_B) = -\frac{1}{4} \to x_A^2 = \frac{1}{4} \xrightarrow{x_A<0} x_A = -\frac{1}{2},\ x_B = \frac{1}{2}$

$f(x_A) = f(-\frac{1}{2}) = -\frac{3}{4}$
$f(x_B) = f(\frac{1}{2}) = -\frac{3}{4}$ $\Big\}$ $= -\frac{3}{4} - \frac{3}{4} = -1.5$

(3)

$m = \frac{18}{3} = 6 \Rightarrow y' = 6 \Rightarrow f'(x) = \frac{-a\times 2}{(2x-1)^2} = 6$

$y = 6x - 9 \to 6x - 9 = \frac{a}{2x-1} \Rightarrow \cancel{12x^2 - 24x + 9 = a} \Rightarrow \frac{-2(4x^2 - 8x + 3)}{4x^2 - 4x + 1} = 6 \Rightarrow$

$f(1) = -\frac{a}{1} = -3 \Rightarrow a = -3$ $\leftarrow (1, -3) =$ نقطه مورد نظر

$\Rightarrow f(0) = -\frac{1}{4}$

(4)

$y' = \frac{1\times(ax+1) - a(x+a)}{(ax+1)^2} = 2 \to \frac{1 - a^2}{(ax+1)^2} = 2 \Rightarrow (a+1)(2a+1) = 0$

$a = -\frac{1}{2}$ ✓

$y = \frac{x - \frac{1}{2}}{-\frac{1}{2}x + 1} = \frac{2x - 1}{-x + 2} \quad x = 1 \quad y = 1$

$a\cdot b = \{\frac{?}{2}\} \quad \to 1 = 2 + b \quad \to y = 2x + b$ در $M(1,1)$

Page:(

۵

$\sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x - \sin x = \cos x \longrightarrow x = \frac{\pi}{4}$

$M(\frac{\pi}{4}, \frac{3\sqrt{2}}{4})$

$f'(x) = \cos x - \frac{1}{2}\sin x \Rightarrow f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{4}$

خط مماس $y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) \xrightarrow{y=0} \boxed{x = \frac{\pi}{4} - 3}$

۶

$f'(x) = 6x^2 - 6x - 12 = 0 \longrightarrow x = -1,\ x = 2$

extrt $\begin{cases} (-1, 8) \\ (2, -19) \end{cases}$ $m_{AB} = -9$

$6x^2 - 6x - 12 = -9 \longrightarrow 6x^2 - 6x - 3 = 0 \xrightarrow{\Delta > 0}$ $\boxed{\text{۲ جواب}}$

۷

$y' = 3kx^2 + 2(k+1)x \longrightarrow y'' = 6kx + 2(k+1) = 0$

$6kx + 2k + 2 = 0 \to x = \frac{-k-1}{3k} < 0$ $\quad -1 \quad 0$ $\quad - \mid + \mid -$ $\quad k > 0$ یا $k < -1$ (I)

عرض نقطه عطف باید مثبت باشد. $f(\frac{-k-1}{3k}) > 0$

$k(\frac{-k-1}{3k})^3 + (k+1)(\frac{-k-1}{3k})^2 > 0 \to k > -1$ (II)

$I \cap II \quad k > 0 \longrightarrow$ همه مقادیر صحیح و مثبت

۸

$M(-1, -4) \to f(-1) = -1 + a - b - 1 = -4 \Rightarrow a - b = -2$

$\to f'(x) = 3x^2 + 2ax + b \longrightarrow -4 = 3 - 2a + b \to b - 2a = -7$

$\frac{a}{b} = \frac{9}{11} \longleftarrow a = 9,\ b = 11$

۹

$f(0) = 4 \to f(x) = x^3 + ax^2 + bx + 4 \quad f'(x) = 3x^2 + 2ax + b \to f'(0) = 0$

$\Rightarrow b = 0 \to f(x) = x^3 + ax^2 + 4 \to f'(x) = x(3x + 2a) \quad f'(x) = 0 \begin{cases} x = 0 \\ x = -\frac{2}{3}a \end{cases}$

$f(-\frac{2}{3}a) = 0 \to (\frac{2a}{3})^3 + a(\frac{-2a}{3})^2 + 4 = 0 \to a = -3$

$x = -\frac{2a}{3} = 2$