نام و نام خانوادگی ....امیرمحمد ابوطالبی........ پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱۹ آذر

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$f(x) = \cos^3(2x) + ax^2 + b$

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \to \frac{\cos^3(2x) + ax^2 + b}{x} \xrightarrow{\text{هم ارزی کسینوس}} \frac{(1-2x^2)^3 + ax^2 + b}{x} \xrightarrow{\text{هم ارزی برنولی}} \frac{1-6x^2+ax^2+b}{x}$

$\to \frac{1+b}{x} + (a-6)x = 0 \to b+1=0 \to \boxed{b=-1}$

$\lim \frac{f'(x)}{x} = 2 \to f'(x) = -6\cos^2(2x)\sin(2x) + 2ax \to \frac{-6\cos^2(2x)\sin(2x) + 2ax}{x}$

$\to -\frac{6\cos^2(2x)\sin(2x)}{x} + 2a \xrightarrow[\cos(2x)\to 1]{\text{هم ارزی سینوس } \sin(2x)\approx 2x} \frac{-6\times 2x}{x} + 2a \to 2a-12=2 \to \boxed{a=7}$

$\boxed{a+b=6}$

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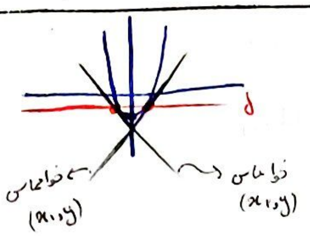

$y = x^2-1 \to y' = 2x$

شیب در نقطه $x_1 = 2x_1$ ← معادله خط مماس: $y = 2x_1 + b_1$

شیب در نقطه $x_2 = 2x_2$ ← معادله خط مماس: $y = 2x_2 + b_2$

شیب خط ۲ خط عمود برهم قرینه و معکوس یکدیگرند

$2x_1 = -\frac{1}{2x_2}$

$\to 4x_1x_2 = -1 \to \boxed{x_1x_2 = -\frac{1}{4}}$ I

چون تابع زوج است پس: $\boxed{x_1+x_2=0}$ II

$\mathrm{I}, \mathrm{II} \to \boxed{x_1 = \frac{1}{2},\ x_2 = -\frac{1}{2}} \to y = x^2-1 \xrightarrow{x=\frac{1}{2}, -\frac{1}{2}} \boxed{y = -\frac{3}{4}} \to \boxed{-\frac{3}{4}\times\frac{2}{2} = -\frac{3}{4}}$

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$f(x) = \frac{a}{2x-1}$

خط مماس بر $f(x)$ از ۲ نقطه $(2.5, 6)$ و $(-0.5, -12)$ می‌گذرد.

پس:

$\frac{\Delta y}{\Delta x} = \frac{-12-6}{-0.5-2.5} = \frac{-18}{-3} = 6$

معادله خط مماس: $y = 6x+b$

شیب تابع $f(x)$ در نقطه مماس برابر ۶ است

$f'(x) = \frac{-2a}{(2x-1)^2} = 6 \to a = -12x^2+12x-3$

$f(x) = \frac{-12x^2+12x-3}{2x-1}$

$y = 6x+b \to x = 2.5$

$y = 15+b = 6 \to \boxed{b=-9}$

$\boxed{y = 6x-9}$

$f(x) = \frac{-3(2x-1)^2}{(2x-1)} = 6x-9 \to -3(2x-1) = 6x-9 \to -6x+3 = 6x-9 \to 12x = 12 \to x=1$

$a = -3(2x-1)^2 \xrightarrow{x=1} \boxed{a=-3} \to f(x) = \frac{-3}{2x-1} \xrightarrow{x=5} \boxed{\frac{-3}{9} = -\frac{1}{3}}$

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$y_1 = 2x+b$

$y_2 = \frac{x+a}{ax+1}$ } مماس در نقطه $(x=1)$

$y_1' = y_2' \to 2 = \frac{(ax+1)-(a)(x+a)}{(ax+1)^2} = \frac{1-a^2}{(ax+1)^2} \xrightarrow{x=1} \frac{1-a^2}{(a+1)^2} = 2$

$\frac{1-a}{a+1} = 2 \to 2a+2 = 1-a \to 3a = -1 \to \boxed{a = -\frac{1}{3}}$

$y_1 = y_2 \to 2+b = \frac{1-\frac{1}{3}}{-\frac{1}{3}+1} \to 2+b = \frac{\frac{2}{3}}{\frac{2}{3}} \to 2+b = 1 \to \boxed{b=-1}$

$a-b = -\frac{1}{3} - (-1) = \boxed{\frac{2}{3}}$

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$f(x) = \sin x + \frac{1}{2}\cos x$

$g(x) = \frac{3}{2}\sin x$ } $\to \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \to \frac{1}{2}\sin x = \frac{1}{2}\cos x \to \sin x = \cos x \to \boxed{x = \frac{\pi}{4}}$

جواب در بازه $[0,\pi]$

$f'(x) = \cos x - \frac{\sin x}{2} \xrightarrow{x=\frac{\pi}{4}} f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{4} \to y = \frac{\sqrt{2}}{4}x + b$

$f(x) = \sin x + \frac{1}{2}\cos x \xrightarrow{x=\frac{\pi}{4}} \frac{3\sqrt{2}}{4} \to y = \frac{\sqrt{2}}{4}x + b \xrightarrow{x=\frac{\pi}{4}} \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}\pi}{16} + b$

$b = \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}\pi}{16} \to 0 = \frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}\pi}{16} \to \boxed{x = 3 - \frac{\pi}{4}}$

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$f(x) = 2x^3 - 3x^2 - 12x + 1 \longrightarrow f'(x) = 6x^2 - 6x - 12 \rightsquigarrow$ $\underset{+ \quad -1 \quad - \quad 2 \quad +}{}$

$A\begin{vmatrix} -1 \\ 8 \end{vmatrix}, B\begin{vmatrix} 2 \\ -19 \end{vmatrix} \longrightarrow \overline{AB} = \frac{\Delta y}{\Delta x} = \frac{8-(-19)}{-1-2} = \frac{27}{-3} = -9$

$f'(x) = -9 \rightarrow 6x^2 - 6x - 12 = -9 \rightarrow 6x^2 - 6x - 3 = 0 \rightsquigarrow \frac{2 \pm 2\sqrt{3}}{4}$

۲ نقطه

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$y = kx^3 + (k+1)x^2 \longrightarrow y' = 3kx^2 + (2k+2)x \longrightarrow \boxed{y'' = 6kx + 2k + 2}$

$\longrightarrow x<0,\ y>0$

$6kx + 2k + 2 = 0 \rightsquigarrow x = \frac{2k+2}{-6k} = \frac{k+1}{-3k}$

$y = k\left(\frac{k+1}{-3k}\right)^3 + (k+1)\left(\frac{k+1}{-3k}\right)^2 \rightarrow y = \frac{2(k+1)^3}{27k^2}$

$\longrightarrow x<0 \rightarrow \frac{k+1}{-3k} < 0 \rightarrow - \ -1 \ + \ 0 \ - \xrightarrow{k<0} \boxed{k<-1}$

$\longrightarrow y>0 \longrightarrow \frac{2(k+1)^3}{27k^2} > 0 \longrightarrow - \ -1 \ + \ 0 \ + \rightsquigarrow -1<k<0$

جمع تعارض

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$y = x^3 + ax^2 + bx - 1 \rightsquigarrow y' = 3x^2 + 2ax + b \longrightarrow x = -1$ $\boxed{3 - 2a + b}$

$y = (-1)^3 + a(-1)^2 + b(-1) - 1 = -4 \rightsquigarrow \underline{a - b = -2}$

$y = (3 - 2a + b)x + c$

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عطف $x = \frac{-b}{3a} = \frac{-a}{3} \rightarrow x = -\frac{a}{3} \rightarrow \frac{-a}{3} = -1 \rightarrow a = 3$

$f(-1) = -4 \rightarrow -1 + 3 - b - 1 = -4 \rightarrow b = -5$

$\frac{a}{b} = \frac{3}{5}$

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$f(x) = x^3 + ax^2 + bx + c \longrightarrow f'(x) = 3x^2 + 2ax + b \xrightarrow{x=0} 0 = 3(0)^2 + 2a(0) + b \rightarrow \boxed{b = 0}$

$f(x) = x^3 + ax^2 + bx + c \xrightarrow{x=0} 4 = (0)^3 + a(0)^2 + b(0) + c \longrightarrow \boxed{c = 4}$

$f(x) = x^3 + ax^2 + bx + c \longrightarrow$

$f'(x) = 3x^2 + 2ax + b$

$0 = 3x^2 + 2ax \rightarrow x(3x + 2a) \rightarrow \boxed{x = -\frac{2a}{3}}$ $\quad x = -\frac{2a}{3} \xrightarrow{a=-3} \boxed{x = 2}$

$f\left(-\frac{2a}{3}\right) = \frac{-8a^3}{27} + \frac{4a^3}{9} + 4 = 0 \rightsquigarrow \frac{4a^3}{27} = -4 \rightsquigarrow \boxed{a = -3}$

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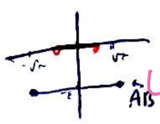
$f(x) = x^4 - 6x^2 + 5 \longrightarrow f'(x) = 4x^3 - 12x \rightsquigarrow x(4x^2 - 12) \rightarrow$ $- \ -\sqrt{3} \ + \ 0 \ - \ \sqrt{3} \ +$

$A\begin{vmatrix} \sqrt{3} \\ -4 \end{vmatrix}, B\begin{vmatrix} -\sqrt{3} \\ -4 \end{vmatrix}$

$f''(x) = 12x^2 - 12 \longrightarrow + \ -1 \ - \ 1 \ + \rightsquigarrow C\begin{vmatrix} 1 \\ 0 \end{vmatrix}, D\begin{vmatrix} -1 \\ 0 \end{vmatrix}$



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