نام و نام خانوادگی ......امیرمحمد ابوطالبی........

پاسخنامه تشریحی تکلیف شماره .... کلاس ....

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$f(x) = \cos^3(2x) + ax^2 + b$

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \rightarrow \frac{\cos^3(2x) + ax^2 + b}{x} \xrightarrow{\text{هم‌ارزی کسینوس}} \frac{(1-2x^2)^3 + ax^2 + b}{x} \xrightarrow{\text{هم‌ارزی بزرگی}} \frac{1-6x^2+ax^2+b}{x}$

$\rightarrow \frac{1+b}{x} + (a-6)x = 0 \rightarrow b+1=0 \rightarrow \boxed{b=-1}$

$\lim \frac{f'(x)}{x} = 2$

$f'(x) = -6\cos^2(2x)\sin(2x) + 2ax \rightarrow \frac{-6\cos^2(2x)\sin(2x) + 2ax}{x}$

$\rightarrow \frac{-6\cos^2(2x)\sin(2x)}{x} + 2a \xrightarrow[\cos(2x) \approx 1]{\text{هم‌ارزی سینوس } \sin(2x) \approx 2x} \frac{-6\times 2x}{x} + 2a \rightarrow 2a - 12 = 2 \rightarrow \boxed{a=7}$

$\boxed{a+b=6}$

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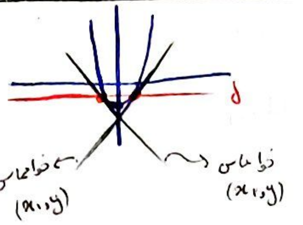

$y = x^2 - 1 \longrightarrow y' = 2x$

شیب در نقطه x_1 = $2x_1$ ← معادله خط مماس: $y = 2x_1 + b_1$

شیب در نقطه x_2 = $2x_2$ ← معادله خط مماس $y = 2x_2 + b_2$

شیب خط ۲ خط عمود بر یکم قرینه و معکوس یکدیگرند

$2x_1 = \frac{-1}{2x_2}$

چون تابع زوج است پس: $\boxed{x_1 + x_2 = 0}$ II

$\rightarrow 4x_1x_2 = -1 \rightarrow \boxed{x_1x_2 = -\frac{1}{4}}$ I

$\text{I}, \text{II} \rightarrow \boxed{x_1 = \frac{1}{2},\ x_2 = -\frac{1}{2}} \rightarrow y = x^2 - 1 \xrightarrow{x=\frac12, -\frac12} \boxed{y = -\frac{3}{4}} \rightarrow \boxed{-\frac{3}{4} \times 2 = -\frac{3}{2}}$

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$f(x) = \frac{a}{2x-1}$

خط مماس بر $f(x)$ از ۲ نقطه $(2.5, 6)$ و $(-0.5, -12)$ می‌گذرد.

پس:

$\frac{\Delta y}{\Delta x} = \frac{-12-6}{-0.5-2.5} = \frac{-18}{-3} = 6$

معادله خط مماس: $y = 6x + b$

شیب تابع $f(x)$ در نقطه مماس برابر ۶ است.

$f'(x) = \frac{-2a}{(2x-1)^2} = 6 \rightarrow a = -12x^2 + 12x - 3$

$y = 6x + b \rightarrow x = 2.5$

$y = 15 + b = 6 \rightarrow \boxed{b = -9}$

$\boxed{y = 6x - 9}$

$f(x) = \frac{-12x^2 + 12x - 3}{2x-1}$

$f(x) = \frac{-3(2x-1)^2}{(2x-1)} = 6x - 9 \rightarrow -3(2x-1) = 6x - 9 \rightarrow -6x + 3 = 6x - 9 \rightarrow 12x = 12 \rightarrow x = 1$

$a = -3(2x-1)^2 \xrightarrow{x=1} \boxed{a = -3} \rightarrow f(x) = \frac{-3}{2x-1} \xrightarrow{x=5} \boxed{\frac{-3}{9} = -\frac{1}{3}}$

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$y_1 = 2x + b$

$y_2 = \frac{x + a}{ax + 1}$

در نقطه $(x=1)$ مماس

$y_1' = y_2' \rightarrow 2 = \frac{(ax+1) - (a)(x+a)}{(ax+1)^2} = \frac{1-a^2}{(ax+1)^2} \xrightarrow{x=1} \frac{1-a^2}{(a+1)^2} = 2$

$\frac{1-a}{a+1} = 2 \rightarrow 2a + 2 = 1 - a \rightarrow 3a = -1 \rightarrow \boxed{a = -\frac{1}{3}}$

$y_1 = y_2 \rightarrow 2 + b = \frac{1 - \frac{1}{3}}{-\frac{1}{3} + 1} \rightarrow 2 + b = \frac{\frac{2}{3}}{\frac{2}{3}} \rightarrow 2 + b = 1 \rightarrow \boxed{b = -1}$

$a - b = -\frac{1}{3} - (-1) = \boxed{\frac{2}{3}}$

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$f(x) = \sin x + \frac{1}{2}\cos x$

$g(x) = \frac{3}{2}\sin x$

$\rightarrow \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \rightarrow \frac{1}{2}\sin x = \frac{1}{2}\cos x \rightarrow \sin x = \cos x \rightarrow \boxed{x = \frac{\pi}{4}}$

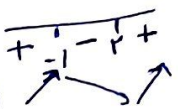
جواب در بازه $[0, \pi]$

$f'(x) = \cos x - \frac{\sin x}{2} \xrightarrow{x = \frac{\pi}{4}} f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{4} \rightarrow y = \frac{\sqrt{2}}{4}x + b$

$f(x) = \sin x + \frac{1}{2}\cos x \xrightarrow{x=\frac{\pi}{4}} \frac{3\sqrt{2}}{4} \rightarrow y = \frac{\sqrt{2}}{4}x + b \xrightarrow{x = \frac{\pi}{4}} \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}\pi}{16} + b$

$b = \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}\pi}{16} \rightarrow 0 = \frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4} - \frac{\sqrt{2}\pi}{16} \rightarrow \boxed{x = 3 - \frac{\pi}{4}}$

$f(x) = 2x^3 - 3x^2 - 12x + 1 \longrightarrow f'(x) = 6x^2 - 6x - 12 \rightsquigarrow$



$A\left|\begin{matrix}-1\\ 8\end{matrix}\right.$, $B\left|\begin{matrix}2\\ -19\end{matrix}\right. \rightsquigarrow \overline{AB} : \frac{\Delta y}{\Delta x} , \frac{8-(-19)}{-1-2} = \frac{27}{-3} = -9$

$f'(x) = -9 \longrightarrow 6x^2 - 6x - 12 = -9 \longrightarrow 6x^2 - 6x - 3 = 0 \rightsquigarrow$

$\frac{2 \pm 2\sqrt{3}}{2}$

2 نقطه

$y = kx^3 + (k+1)x^2 \longrightarrow y' = 3kx^2 + 2(k+1)x \longrightarrow \boxed{y'' = 6kx + 2k + 2}$

$\longrightarrow x<0 , y>0$

$6kx + 2k + 2 = 0 \rightsquigarrow x = \frac{-2k-2}{6k} = \frac{k+1}{-3k}$

$y = k\left(\frac{k+1}{-3k}\right)^3 + (k+1)\left(\frac{k+1}{-3k}\right)^2 \rightarrow y = \frac{(k+1)^3}{-27k^3}$

$\longrightarrow x<0 \rightarrow \frac{k+1}{-3k} < 0 \longrightarrow$  $\rightsquigarrow k<0$ $\boxed{k<-1}$

$\longrightarrow y>0 \longrightarrow \frac{-(k+1)^3}{27k^3} > 0 \longrightarrow$  $\rightsquigarrow -1 < k < 0$ قبیح نقطه

$y = x^3 + ax^2 + bx - 1 \rightsquigarrow y' = 3x^2 + 2ax + b \longrightarrow x = -1$ $\boxed{3 - 2a + b}$

$y = (-1)^3 + a(-1)^2 + b(-1) - 1 \xrightarrow{y=-2} \boxed{a - b = -2}$

$y = (3 - 2a + b)x + c$

$f(x) = x^3 + ax^2 + bx + c \longrightarrow f'(x) = 3x^2 + 2ax + b \xrightarrow{x=0} 0 = 3(0)^2 + 2a(0) + b \rightarrow \boxed{b = 0}$

$f(x) = x^3 + ax^2 + bx + c \xrightarrow{x=0} 2 = (0)^3 + a(0)^2 + b(0) + c \longrightarrow \boxed{c = 2}$

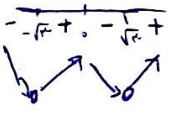
$f(x) = x^3 + ax^2 + bx + c \longrightarrow$

$f'(x) = 3x^2 + 2ax + b$

$0 = 3x^2 + 2ax \rightarrow x(3x + 2a) \rightarrow$ $\boxed{x = -\frac{2a}{3}}$. $x = -\frac{2a}{3} \xrightarrow{a=-3} x = 2$

$f\left(-\frac{2a}{3}\right) = \frac{-8a^3}{27} + \frac{4a^3}{9} + c = 0 \rightsquigarrow \frac{4a^3}{27} = -4 \rightsquigarrow \boxed{a = -3}$

$f(x) = x^4 - 6x^2 + 5 \longrightarrow f'(x) = 4x^3 - 12x \rightsquigarrow x(4x^2 - 12) \rightarrow$



$A\left|\begin{matrix}\sqrt{3}\\ -4\end{matrix}\right.$, $B\left|\begin{matrix}-\sqrt{3}\\ -4\end{matrix}\right.$

$f''(x) = 12x^2 - 12 \longrightarrow$ $\rightsquigarrow C\left|\begin{matrix}1\\ 0\end{matrix}\right. , D\left|\begin{matrix}-1\\ 0\end{matrix}\right.$