۱)

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \rightarrow f'_+(0) = 0$

$\lim_{x\to 0^-} \frac{f(x)}{x} = 2 \rightarrow f'_-(0) = 2$

$f(0) = 1+b = 0 \rightarrow b = -1$

$f'(x) = 3\cos^2 x(-2\sin x)(-\sin x) + 2ax$

$f''(x) = 6\cos x(-\sin x)(-\sin x)(-\sin x) + 3\cos^2 x(-\cos x) + 2a \Big|_{x=0} \rightarrow -1 + 2a = 2 \rightarrow a = 1$

$1 + (-1) = 0$ ✓ ج

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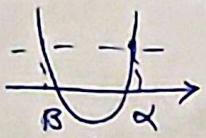
۲)

$y' = 2x$



$2\alpha \times 2\beta = -1 \rightarrow \alpha\beta = -\frac{1}{4}$

$\alpha = -\beta \qquad -\alpha^2 = -\frac{1}{4} \rightarrow \alpha = \frac{1}{2}$

$\beta = -\frac{1}{2}$

$\left(\frac{1}{2}-1\right)^2 = \frac{1}{4}$ ج

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۳)

$m = \frac{9+12}{11-6} = \frac{21}{3} = 7 \rightarrow d: 9(x-2) = y-9 \rightarrow y = 9x-9$

$f(1) = d \xrightarrow{\Delta=0} \frac{a}{2m-1} = 9m-9 \rightarrow a = 18m^2 - 36m + 9 \rightarrow 18m^2 - 36m + 9 - a$

$36^2 - 18(9-a) = 0 \rightarrow a = -2$

$f(x) = \frac{-2}{2m-1} \Big|_{x=0} \rightarrow \frac{-2}{-2} = \frac{1}{2}$ ج

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۴)

$y' = \frac{1-ax^2}{(ax+1)^2} \Big|_{x=1} \rightarrow \frac{1-a^2}{(a+1)^2} = 0 \quad a = \frac{1}{-1}, \; a = -\frac{1}{3}$

$\rightarrow y = \frac{x-1}{1-x} = -1 \rightarrow$ تقاطع x

$\rightarrow y = \frac{x-\frac{1}{3}}{-\frac{1}{3}x+1} \Big|_{x=1} \rightarrow y = 1$

$1 = 2(1) + b \rightarrow b = -1$

$-\frac{1}{3} + 1 = \frac{2}{3}$ ج

۶)

$f=g \to \frac{1}{4}\sin^4 x = \sin x + \frac{1}{4}\cos x \to \frac{1}{4}\sin^4 x = \frac{1}{4}\cos x$ $x \in [0,\pi]$ $x=\frac{\pi}{3}$

$f'(x) = \cos x - \frac{1}{4}\sin^3 x \cdot \cos x \Big|_{x=\frac{\pi}{3}} \to f'(\frac{\pi}{3}) = \frac{3}{16}$

$y - \frac{3\sqrt{3}}{8} = \frac{3}{16}(x-\frac{\pi}{3}) \to y = \frac{3}{16}x + \frac{3\sqrt{3}}{8} - \frac{\pi}{16}$

$f(\frac{\pi}{3}) = \frac{3\sqrt{3}}{8}$

۷)

$f(x) = 9x^3 - 3x^2 - 12x + 1 = 6(x^3 - x^2 + 2x)$

$6(x-2)(x+1) = 0$

x	-1		2	
f'	$+$	$-$	0	$+$
f	↗	max	↘	min

$f(-1) = -1-3+12+1 = 9$

$f(2) = 8-12-24+1 = -27$

$m_{AB} = \frac{9+27}{-1-2} = -12$

$f(x) = -27 \to 9x^3 - 3x^2 - 12x = -27 \to 3x^3 - x^2 - 4x + 9 = 0$

$(x+3)(3x^2-10x+3) = 0$

$\Delta > 0$ → دو ریشه

۸)

$x = \frac{-b}{2a} = \frac{-(k-1)}{2k}$

	-1		0	
	$+$	$-$	$+$	

$k<0 \to k<-1$

$f(x) > 0$ $y = x^2(kx+k+1)\Big|_{x=\frac{-k-1}{2k}}$

$\left(\frac{k+1}{2k}\right)^2\left(\frac{-k-1}{2}+k+1\right)$

$\frac{k+1}{2k} > 0$ → $k<-1$

$k<-1$

۹)

$f' = 3x^2 + 2ax + b$

$f''(-1) = 0 \to -6+2a=0 \to a=3$

$f'(-1) = 0 \to 3-6+b=0 \to b=3$

$\frac{b}{a} = 1$

۱۰)

$f(0) = 2 \to c = 2$

$f'(0) = 0 \to b = 0$

$f'(x) = x(3x^2+3ax+2) \to \min$ → $f'(-2) = 0$

$12 - 6a + 2 = 0 \to a = \frac{7}{3}$

$f(-2) = -8 + 14 - 14 + 2 = -6$

۱۱)

$f(x) = x^3 - 3x^2 - 9x + 1$

$f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$

$f(-1) = -1 - 3 + 9 + 1 = 6$

$f(3) = 27 - 27 - 27 + 1 = -26$

$m_{AB} = \frac{-32}{4} = -8$

x	$-\infty$		-1		3	
f'	$+$	0	$-$	0	$+$	
f	↗		↘		↗	

$\min \to x = 3$

$m_{OD} = 0$

$m_{AB} = \frac{-32}{4} = -8$

$m_{OD} \cdot m_{AB} = 0$ → عمودند

توجه: کلافکی