امین جلالی

(1)

$y = \frac{x^2+x+2}{x-1} \Rightarrow yx - y = x^2+x+2 \Rightarrow x^2+x-xy+y+2 = x^2+(1-y)x+(y+2)=0$

$\Rightarrow \Delta \geq 0 \Rightarrow (1-y)^2 - 4(y+2) = 1+y^2-2y-4y-8 = y^2-6y-7 \Rightarrow (y-7)(y+1)$

$\begin{array}{ccccc} & -1 & & 7 & \\ + & | & - & | & + \end{array} \Rightarrow R_f = (-\infty,-1] \cup [7,+\infty)$

(2)

الف) $-\frac{b}{2a} = \frac{5}{4} \Rightarrow y = 2\left(\frac{25}{16}\right) - \frac{25}{4} + 1 \Rightarrow \frac{25-50+8}{8} = \frac{-17}{8} \Rightarrow$ 

$\Rightarrow R_f = \left[-\frac{17}{8}, +\infty\right)$

ب) $-x^2+6x-5 \Rightarrow -\frac{b}{2a} = 3 \Rightarrow y = -9+18-5 = 4 \Rightarrow R = (-\infty,4] \xRightarrow{\sqrt{\ }} [0,2] \leftarrow R_f$

ج) $\longrightarrow$ حداکثر توان فرد است $\Longrightarrow R_f = \sqrt{(-\infty,+\infty)} = [0,+\infty)$

د) $\longrightarrow$ حداکثر توان فرد $\Rightarrow D_f = (0,+\infty) \Rightarrow R_f = \mathbb{R}$

(3)

الف) $R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_f = \mathbb{R} - \left\{\frac{3}{2}\right\}$

ب) $R_f = \sqrt{\mathbb{R}} - \left\{\frac{a}{c}\right\} \Rightarrow R_f = [0,+\infty) - \{4\}$

ج) $x^2_{min} = 0 \ , \ x^2_{max} = +\infty \Rightarrow R_f = \left[\frac{4}{\sqrt{3}}, +\infty\right)$

د) $x^2+4 + \frac{1}{x^2+4} - 4 \Rightarrow \underbrace{x^2+4}_{a} + \underbrace{\frac{1}{x^2+4}}_{\frac{1}{a}} - 4 \Rightarrow a + \frac{1}{a} - 4 \Rightarrow R_f = [-2,+\infty)$

$a + \frac{1}{a} \geq 2$

(4)

روش ۱) $yx^2 - y = 3x^2+4 \Rightarrow 3x^2 - yx^2 = -y-4 \Rightarrow x^2 = \frac{y+4}{y-3} \Rightarrow x = \sqrt{\frac{y+4}{y-3}}$

$\Rightarrow \frac{y+4}{y-3} \geq 0 \Rightarrow \begin{array}{ccccc} & -4 & & 3 & \\ + & | & - & | & + \end{array} \Rightarrow R_f = (-\infty,-4] \cup (3,+\infty)$

روش ۲) $\begin{array}{l} \min x^2 = 0 \\ \max x^2 = +\infty \end{array} \Rightarrow \begin{array}{l} x^2 = 0 \rightarrow -4 \\ x^2 = +\infty \rightarrow 3 \end{array}$ چون مخرج صفر می‌شود بیرون بازه $\rightarrow (-\infty,-4] \cup (3,+\infty)$

(5)

روش ۱) $3y+1 = 2\sin x - y\sin x \Rightarrow \sin x = \frac{3y+1}{2-y} \Rightarrow -1 \leq \frac{3y+1}{2-y} \leq 1 \Rightarrow -\frac{3}{2} \leq y \leq \frac{1}{4}$

روش ۲) $\begin{array}{l} \min \sin x = -1 \\ \max \sin x = 1 \end{array} \Rightarrow \begin{array}{l} -1 = \frac{-2-1}{-1+3} = -\frac{3}{2} \\ 1 = \frac{2-1}{1+3} = \frac{1}{4} \end{array} \Rightarrow R_f = \left[-\frac{3}{2}, \frac{1}{4}\right]$

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$D(f) \Rightarrow$ مخرج $\neq 0 \Rightarrow \Delta_{\text{مخرج}} < 0 \rightarrow$ ریشه ندارد $\Rightarrow D_f = \mathbb{R}$

$\dfrac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2+\frac{3}{4}} \Rightarrow (x-\frac{1}{2})_{min}=0 \quad (x-\frac{1}{2})_{max}=+\infty \Rightarrow R_{(f)} = [0, +\infty)$

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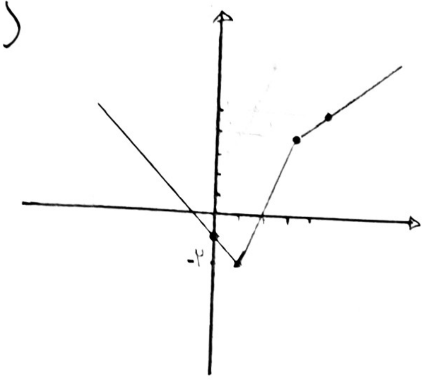
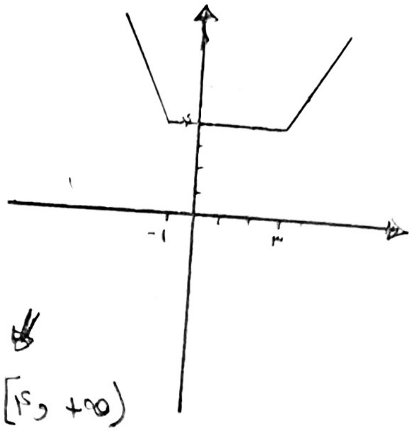
الف)



$\Rightarrow R_{(f)} = [-2, +\infty)$



$R_{(f)} = [4, +\infty)$

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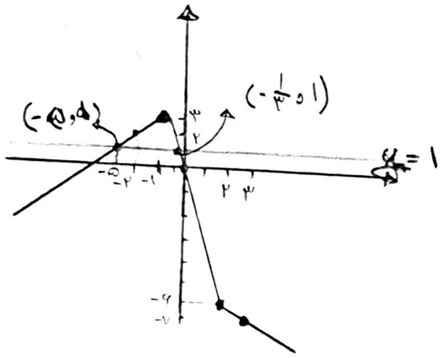

$\Rightarrow$ جواب $= \mathbb{R} - (-5, -\frac{1}{3})$

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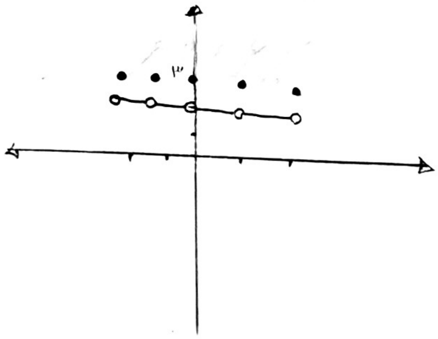
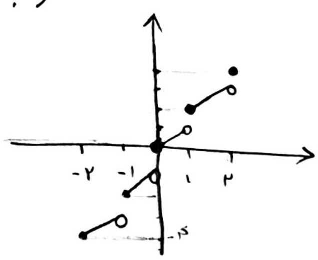
الف) $y = [x] + [-x] + 3 \rightarrow$ نمودار $x+[-x]$ ۳ واحد به سمت بالا



$\Rightarrow R_{(f)} = \{3, 2\}$

ب)



$R_{(f)} = [-4, 4]$

۱۰

الف) $\sin x = -1 \rightarrow 1$

$\sin = 1 \rightarrow 1$

$-\frac{b}{2a} = \frac{1}{2} \rightarrow \frac{3}{4}$

$\Rightarrow R_{(f)} = [\frac{3}{4}, 1]$

ب) $\sin^2 = 0 \rightarrow 1$

$\sin^2 = 1 \rightarrow 3$

$\sin^2 = -\frac{1}{2} \rightarrow$ غیرقابل قبول

$\Rightarrow R_{(f)} = [1, 3]$