تکلیف ۳:

مهراد سامع

۱) $yx-y=x^2+x+2 \Rightarrow x^2+(1-y)x+(2+y)=0 \Rightarrow \Delta \geq 0$

$y^2-6y-7 \geq 0 \Rightarrow y_0=\frac{6\pm 8}{2}=-1,7 \Rightarrow$ $\begin{array}{ccccc} & -1 & & 7 & \\ + & | & - & | & + \end{array}$

$\Rightarrow R_f=(-\infty,-1] \cup [7,+\infty)$

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۲) الف) $a>0 \Rightarrow min=\frac{-\Delta}{4a} \Rightarrow \frac{-17}{8} \Rightarrow R_f=[\frac{-17}{8},+\infty)$

ب) $a<0 \Rightarrow max=\frac{-\Delta}{4a} \Rightarrow \frac{-16}{-4}=4 \Rightarrow R_f=[0,2]$

ج) $\sqrt{x^2-3x+2+4x} \geq 0 \Rightarrow R_f=[0,+\infty)$

د) $x^3-6x^2+11x+6=R \Rightarrow \log_{10} x^3-6x^2+11x+6 \rightarrow R_f=R$

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۳) الف) $y=\frac{3x+1}{2x-6} \rightarrow -\frac{-6y-1}{2y-3}=\frac{6y+1}{2y-3} \Rightarrow R_f=R-\{\frac{3}{2}\}$

ب) $y=\sqrt{\frac{4x+1}{x-2}} \Rightarrow x=\frac{2y+1}{y-4} \Rightarrow R_f=[0,+\infty)-\{2\}$

ج) $\frac{x^2+4}{\sqrt{x^2+3}}=\sqrt{x^2+3}+\frac{1}{\sqrt{x^2+3}} \Rightarrow R_f=[\frac{4}{\sqrt{3}},+\infty)$

د) $x^2+\frac{1}{x^2+4} \xrightarrow{+4,-4} x^2+4+\frac{1}{x^2+4}-4 \Rightarrow R_f=[\frac{1}{4},+\infty)$

۴)

① $yx^2 - y = 3x^2 + 4 \Rightarrow x^2 = \frac{-(4+y)}{3-y} \Rightarrow x = \sqrt{\frac{-(4+y)}{3-y}}$

$-4 \quad 3$

$+ \mid - \mid + \Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

②

۵)

① $y = \frac{2\sin x - 1}{\sin x + 3} \Rightarrow \sin x = \frac{-3y-1}{y-2} \Rightarrow -1 \leq \sin x \leq 1$

$\Rightarrow -3y - 1 = y - 2 \Rightarrow y = \frac{1}{4} \quad , \quad -3y - 1 = -y + 2 \Rightarrow y = -\frac{3}{2} \Rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$

②

۶)

1. $D_f = R - \{$ریشه مخرج$\}$ ⇐ مخرج ریشه ندارد ⇐ $D_f = R$

2. $yx^2 - yx + y = x^2 - x + \frac{1}{4} \Rightarrow (y-1)x^2 + (1-y)x + (y - \frac{1}{4}) = 0$

$\Rightarrow \Delta \geq 0 \Rightarrow y^2 - 2y + 1 - y^2 + \cancel{\frac{5}{4}}y - \frac{1}{4} \Rightarrow -\frac{3}{4}y + \frac{3}{4} \geq 0$

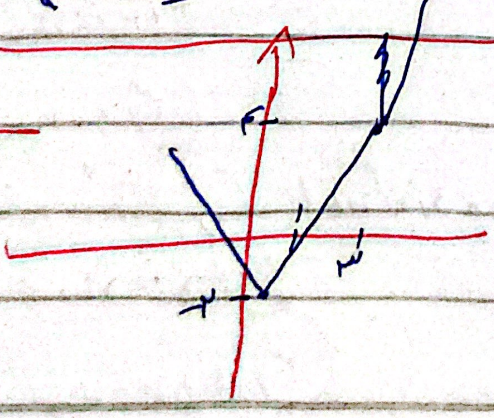
$\Rightarrow \frac{3}{4}y \leq \frac{3}{4} \Rightarrow y \leq 1 \Rightarrow D_f = (-\infty, 1]$

۷)

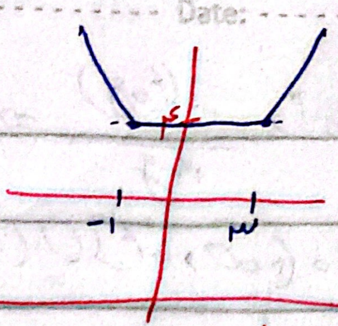
الف ⇐ $\quad 1 \quad 3$

$-x-1 \mid 2x-5 \mid x+1$

$\Rightarrow D_f = [2, +\infty)$

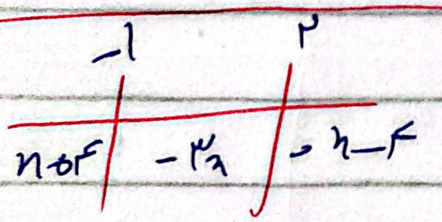


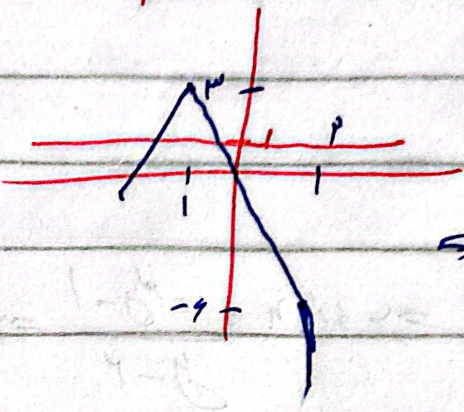
ب) گلدانی ⇒



$\Rightarrow D_f=[4,+\infty)$

۸)

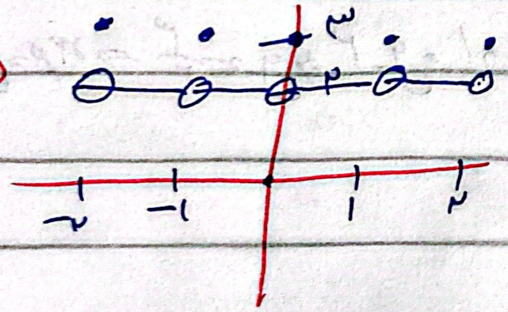




$\Rightarrow$ جواب $=\mathbb{R}-(3,-\frac{1}{3})$

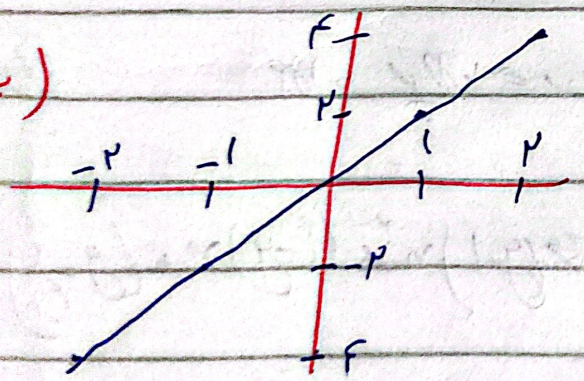
~~⇒~~

۹) الف) ⇒



$\Rightarrow R_f=\{2,3\}$

ب)



$D_f=[-4,4]$

۱۰) الف) $\sin^2 x-\sin x+1$

① $\sin x=1 \Rightarrow 1$

② $\sin x=-1 \Rightarrow 3$

③ $\sin x=\frac{1}{2} \Rightarrow \frac{3}{4}$

$R_f=[\frac{3}{4},3]$

ب) $\sin^4 x+\sin^2 x+1$

① $\sin^2 x=1 \Rightarrow 3$

② $\sin x=0 \Rightarrow 1$

③ $\sin x=-\frac{1}{2} \Rightarrow \frac{21}{16}$

$R_f=[1,3]$