

۱) $y = \frac{x^2 + x + 2}{x - 1} \Rightarrow y(x - 1) = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + 2 + y = 0$
 استفاده از تبیین!
 $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{y - 1 \pm \sqrt{(1 - y)^2 - 4(2 + y)}}{2}$
 $1 + y^2 - 2y - 8 - 4y \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow (y - 7)(y + 1) \geq 0$
 $R_f = R - (-1, 7) \checkmark$

۲) $y = 10x^2 - 10x + 1$
 کد را در ۲ آزمون باک
 $\Delta = b^2 - 4ac = 100 - 40 = 60$
 $y = \frac{10 \pm \sqrt{60}}{20} = \frac{10 \pm 2\sqrt{15}}{20} = \frac{5 \pm \sqrt{15}}{10}$
 ۱) $y = \sqrt{x^2 + 4x + 5} \Rightarrow x^2 + 4x + 5 = y^2 \Rightarrow x^2 + 4x + 5 - y^2 = 0$
 $\Delta = 16 - 4(5 - y^2) = 4y^2 - 4 \geq 0 \Rightarrow y^2 \geq 1 \Rightarrow y \leq -1 \text{ or } y \geq 1$
 ۲) $R_f = [0, 1) \cup (1, \infty) \checkmark$
 ۳) $R_f = (-\infty, +\infty) \checkmark$

۳) $y = \frac{x^2 + 1}{x^2 - 2x} \Rightarrow y(x^2 - 2x) = x^2 + 1 \Rightarrow (y - 1)x^2 - 2yx - 1 = 0$
 $\Delta = 4y^2 + 4(y - 1) = 4y^2 + 4y - 4 = 4(y^2 + y - 1) \geq 0$
 $y^2 + y - 1 \geq 0 \Rightarrow y \leq \frac{-1 - \sqrt{5}}{2} \text{ or } y \geq \frac{-1 + \sqrt{5}}{2}$
 $R = (-\infty, \frac{-1 - \sqrt{5}}{2}] \cup [\frac{-1 + \sqrt{5}}{2}, \infty)$
 ۱) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow R_f = [0, \infty) \cup (-\sqrt{2}, \sqrt{2})$
 ۲) $\sqrt{x^2 + 4} = y \Rightarrow x^2 + 4 = y^2 \Rightarrow x^2 = y^2 - 4 \geq 0 \Rightarrow y^2 \geq 4 \Rightarrow y \leq -2 \text{ or } y \geq 2$
 $R_f = (-\infty, -2] \cup [2, \infty)$
 ۳) $y = x^2 + \frac{1}{x^2 + 4} \Rightarrow (x^2 + 4)y = x^2 + 1 \Rightarrow (y - 1)x^2 + 4y - 1 = 0$
 $\Delta = 16y^2 - 4(y - 1) = 16y^2 - 4y + 4 \geq 0$
 $4y^2 - y + 1 \geq 0$
 $R_f = (-\infty, \infty) \checkmark$

۴) $y x^2 - y = x^2 + 1 \Rightarrow (y - 1)x^2 - y - 1 = 0$
 $\Delta = 4y^2 + 4(y - 1) = 4y^2 + 4y - 4 = 4(y^2 + y - 1) \geq 0$
 $y^2 + y - 1 \geq 0 \Rightarrow y \leq \frac{-1 - \sqrt{5}}{2} \text{ or } y \geq \frac{-1 + \sqrt{5}}{2}$
 $R = (-\infty, \frac{-1 - \sqrt{5}}{2}] \cup [\frac{-1 + \sqrt{5}}{2}, \infty)$
 $x^2 = 0 \Rightarrow y = \frac{1}{-1} = -1$
 $x^2 = +\infty \Rightarrow y = 1$
 $R_f = R - (-1, 1) \checkmark$

۵) $y \sin x + 2y = 2 \sin x - 1 \Rightarrow y(\sin x + 2) = 2 \sin x - 1$
 $\sin x = \frac{2 - y}{1 + y}$
 $-1 \leq \frac{2 - y}{1 + y} \leq 1$
 $\sin x = 1 \Rightarrow y = \frac{2 - 1}{1 + 1} = \frac{1}{2}$
 $\sin x = -1 \Rightarrow y = \frac{2 - (-1)}{1 + (-1)} = \frac{3}{0} \Rightarrow y \neq -1$
 $R_f = [-\frac{1}{2}, \frac{1}{2}] \checkmark$

$$u^2 - u = a$$

$$u^2 - u \Rightarrow -\frac{b}{2a} = \frac{1}{2}$$

$$f(u) = \frac{u + \frac{1}{2}}{u + 1}$$

$$u = -\frac{1}{2} \Rightarrow f(u) = \dots$$

$$R_f = (0, 1)$$

$$u^2 - u \neq -1 \Rightarrow u^2 - u + 1 \neq 0$$

$$\Delta < 0 \Rightarrow 1 - 4 < 0$$

$$D_f = \mathbb{R}$$

$$u = 1 \Rightarrow y = 2$$

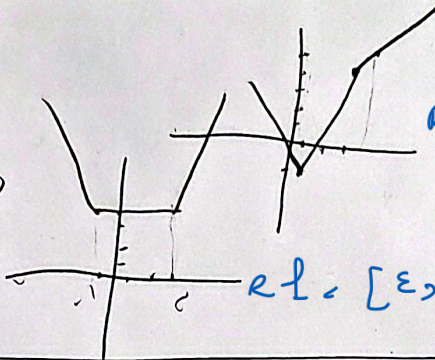
$$y = 2$$

$$u = 0 \Rightarrow y = -1$$

$$y = -1$$

$$u = 2$$

$$y = 0$$



$$R_f = [-2, +\infty)$$

$$R_f = [\varepsilon, +\infty)$$

$$u = 2 \Rightarrow y = -4$$

$$u = -1 \Rightarrow y = 2$$

$$u = 1 \Rightarrow y = -2$$

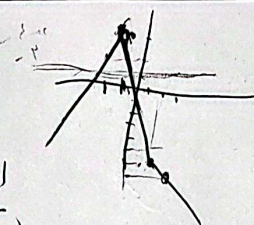
$$u = -2 \Rightarrow y = 1$$

$$y = -u + 1 + 2u + 1 \Rightarrow u + 2 = 1$$

$$u = -1$$

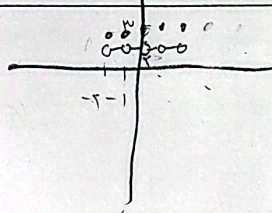
$$-2u + 1 - u + 1 \Rightarrow -3u + 2 = 0$$

$$u = 1 \Rightarrow y = -\frac{1}{2}$$



$$R_f = \mathbb{R} - \left(-\frac{1}{2}, -1\right)$$

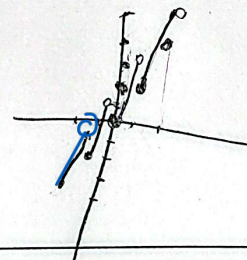
$$y = [u] + [1 - u] + 2$$



$$R_f = \{1, 2\}$$

$$y = 2u$$

$$y = 2u \Rightarrow u = \frac{y}{2}$$



$$R_f = [2, 0)$$

$$y = \sin u - \sin u + 1 = 1$$

$$\sin u = 1 \Rightarrow u = \frac{\pi}{2}$$

$$\sin u = -1 \Rightarrow u = \frac{3\pi}{2}$$

$$\sin u = \frac{\sqrt{2}}{2} \Rightarrow u = \frac{\pi}{4}$$

$$\sin u = -\frac{\sqrt{2}}{2} \Rightarrow u = \frac{5\pi}{4}$$

$$R_f = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$y = \sin u + \sin u + 1 = 2\sin u + 1$$

$$\sin u = 0 \Rightarrow y = 1$$

$$\sin u = 1 \Rightarrow y = 2$$

$$R_f = [1, 2]$$