-۱
$y=\frac{x^2+x+2}{x-1} \to x^2+(1-y)x+y+2=0 \to x=\frac{y-1\pm\sqrt{y^2+1-2y-4y-8}}{2}$

$\to \Delta\geq 0 \to y^2-6y-7\geq 0 \to (y-7)(y+1)\geq 0 \to \begin{array}{ccccc} & -1 & & 7 & \\ + & 0 & - & 0 & + \end{array} \to R_f=(-\infty,-1]\cup[7,+\infty)$

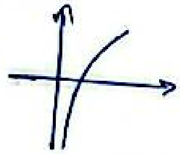
-۲

الف) $y=2x^2-5x+1 \to Min \left| \begin{array}{l} \frac{5}{4} \\ \frac{25}{8}-\frac{25}{4}+1=-\frac{17}{8} \end{array}\right. \to R_f=[-\frac{17}{8},+\infty)$

ب) $Max \left| \begin{array}{l} 3 \\ 4 \end{array}\right. \xrightarrow{\sqrt{\ }} R_f=[0,2]$

کاملا ریجی

ج) $y=\sqrt{x^5-2x^2+4x+1} \longrightarrow$ بزرگترین توان فرد است $\to R_f=\mathbb{R} \xrightarrow{\sqrt{\ }} R_f=[0,+\infty)$

د) $x^5-6x^2+7x+3 \longrightarrow$ بزرگترین توان فرد است $\to \mathbb{R} \to \log\underbrace{x^5-6x^2+7x+4}_{K,\ K\in\mathbb{R}} \to$  $\to R_f=\mathbb{R}$

-۳

الف) $y=\frac{3x+1}{2x-4} \xrightarrow{\text{هموگرافیک}} R_f=\mathbb{R}-\{\frac{3}{2}\}$

ب) $y=\sqrt{\frac{4x+1}{x-2}} \to R_f=\mathbb{R}-\{4\} \xrightarrow{\sqrt{\ }} R_f=[0,+\infty)-\{2\}$

ج) $\frac{x^2+4}{\sqrt{x^2+3}}=\frac{x^2+3+1}{\sqrt{x^2+3}}=\underbrace{\sqrt{x^2+3}}_{\text{همواره مثبت}}+\frac{1}{\sqrt{x^2+3}} \to R_f=[\sqrt{3}+\frac{1}{\sqrt{3}},+\infty)=[\frac{4\sqrt{3}}{3},+\infty)$

حداقل مقدار تابع به ازای $x=0$ است ← می‌باشد

د) $y=x^2+\frac{1}{x^2+4}=\underbrace{x^2+4+\frac{1}{x^2+4}}-4 \to R_f=[4+\frac{1}{4},+\infty) \xrightarrow{-4} R_f=[\frac{1}{4},+\infty)$

-۴

$y=\frac{3x^2+4}{x^2-1} \to x^2=-\frac{-y-4}{y-3}=\frac{y+4}{y-3}=x^2 \to x=\pm\sqrt{\frac{y+4}{y-3}} \to \begin{array}{ccccc} & -4 & & 3 & \\ + & 0 & - & | & + \end{array}$

$\to R_f=(-\infty,-4]\cup(3,+\infty)$

-۵

$y=\frac{2\sin x-1}{\sin x+3} \to \sin x=\frac{-(3y+1)}{y-2}=\frac{-3y-1}{y-2}=\sin x \xrightarrow{-1\leq \sin x\leq 1} -1\leq\frac{-3y-1}{y-2}\leq 1$

$\frac{-3y-1}{y-2}\leq 1 \to \frac{-3y-1-y+2}{y-2}\leq 0 \to \frac{-4y+1}{y-2}\leq 0 \to \begin{array}{ccccc} & \frac{1}{4} & & 2 & \\ - & 0 & + & | & - \end{array} \to (-\infty,\frac{1}{4}]\cup(2,+\infty)$

$-1\leq\frac{-3y-1}{y-2} \to \frac{-3y-1+y-2}{y-2}\geq 0 \to \frac{-2y-3}{y-2}\geq 0 \to \begin{array}{ccc} -\frac{3}{2} & & 2 \end{array} \to [-\frac{3}{2},2)$

اشتراک $\to R_f=[-\frac{3}{2},\frac{1}{4}]$

CamScanner

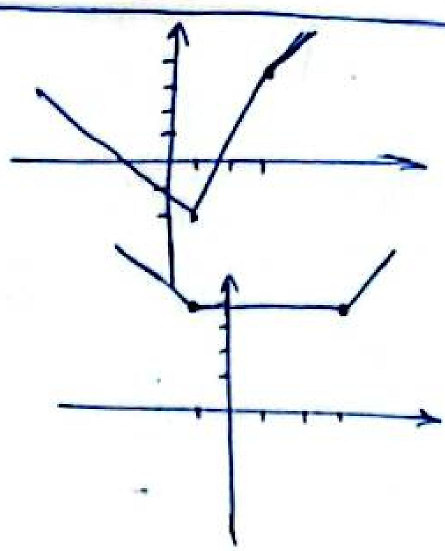
$x^2-x+1 \neq 0 \rightarrow \Delta<0 \rightarrow D_f=\mathbb{R}$

۶- $f(x)=\dfrac{x^2-x+\frac{1}{4}}{x^2-x+1} \rightarrow R_f: \dfrac{x^2-x+1-\frac{3}{4}}{x^2-x+1}=1-\dfrac{3}{4x^2-4x+4}$

مربع کامل مثبت

$\rightarrow \underbrace{4x^2-4x+4}_{a}\geq 3 \rightarrow 1-\frac{3}{a} \rightarrow 0\leq 1-\frac{3}{a} \rightarrow R_f=[0,+\infty)$

۷- الف) $y=|2x-2|-|x-3| = \begin{cases} x+1 & x\geq 3\\ 3x-5 & 1\leq x\leq 3\\ -x-1 & x\leq 1\end{cases} \rightarrow$

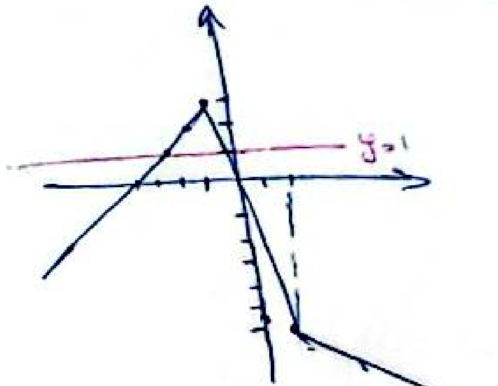


$R_f=[-2,+\infty)$

ب) $|x-3|+|x+1| \rightarrow \begin{cases} 2x-2 & x\geq 3\\ 4 & -1\leq x\leq 3\\ -2x+2 & x\leq -1\end{cases} \rightarrow$

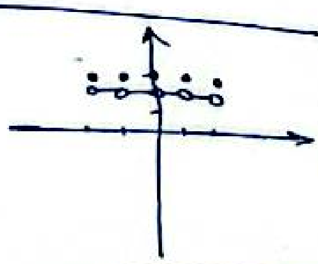
$R_f=[4,+\infty)$

۸- $y=|x-2|-|2x+2| \rightarrow \begin{cases} -x-4 & x\geq 2\\ -3x & -1\leq x\leq 2\\ x+4 & x\leq -1\end{cases}$



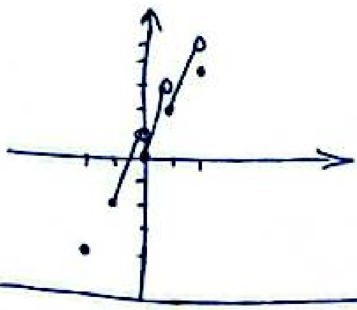
$\rightarrow x\in(-\infty,-3]\cup[-\frac{1}{3},+\infty)$

۹- الف) $[x]+[3-x]=[x]+[-x]+3$:



$R_f=\{2,3\}$

ب) $3x-[x]$:



$R_f=[-2,a)\cup\{-2\}$

۱۰- الف) $y=S^2-S+1$

$\begin{cases} S=1 \rightarrow y=1\\ S=-1 \rightarrow y=3\\ -\frac{b}{2a} \rightarrow \frac{1}{2} \rightarrow y=\frac{3}{4}\end{cases} \quad R_f=[\frac{3}{4},3]$

ب) $y=S^4+S^2+1=(S^2)^2+S^2+1$

$\begin{cases} S^2=1 \rightarrow y=3\\ S^2=0 \rightarrow y=1\\ -\frac{b}{2a} \rightarrow -\frac{1}{2} \rightarrow S^2=-\frac{1}{2} \times \end{cases} \quad R_f=[1,3]$