

$$y = \frac{x^2 + x + 2}{x-1} \rightarrow yx - y = x^2 + x + 2 \rightarrow 0 = x^2 + (1-y)x + y+2$$

$$\rightarrow x = \frac{y-1 \pm \sqrt{(1-y)^2 - 4(y+2)}}{2} \rightarrow \Delta \geq 0 \rightarrow y^2 - 2y + 1 - 4y - 8 \geq 0$$

$$\rightarrow y^2 - 6y - 7 \geq 0 \rightarrow \begin{matrix} + & | & - & | & + \end{matrix} \rightarrow R_y = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 5x + 1 \rightarrow \text{ext} = \left(\frac{5}{4}, -\frac{17}{8}\right) \rightarrow R_y = \left[-\frac{17}{8}, +\infty\right)$$

$a \geq 0$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 5} \rightarrow y' = -x^2 + 4x - 5 \rightarrow \text{ext} = (2, 4)$$

$a \leq 0 \rightarrow R_{y'} = (-\infty, 4]$

$$\rightarrow \sqrt{y'} = y \rightarrow R_y = [0, 4]$$

$$\text{ج) } y = \sqrt{x^4 - 3x^2 + 4x + 1} \rightarrow y' = x^4 - 3x^2 + 4x + 1 \rightarrow R_{y'} = \mathbb{R} \rightarrow y = \sqrt{y'}$$

$$\rightarrow R_y = [0, +\infty)$$

$$\text{د) } \log(x^3 - 4x^2 + 7x + 4) \rightarrow y' = x^3 - 4x^2 + 7x + 4 = (0, 1] \rightarrow \log(y') = (-\infty, 0]$$

$$y' = x^3 - 4x^2 + 7x + 4 = [1, +\infty) \rightarrow \log(y') = [0, +\infty)$$

$$R_{\log(x^3 - 4x^2 + 7x + 4)} = (-\infty, +\infty)$$

$$\text{الف) } y = \frac{3x+1}{2x-5} \rightarrow R_y = \mathbb{R} - \left\{\frac{3}{2}\right\} \rightarrow R_y = \mathbb{R} - \left\{\frac{3}{2}\right\}$$

$$\text{ب) } y = \sqrt{\frac{4x+1}{x-2}} \rightarrow y' = \frac{4x+1}{x-2} \rightarrow R_{y'} = \mathbb{R} - \{2\} \rightarrow y = \sqrt{y'} \rightarrow R_y = [0, +\infty) - \left\{\frac{1}{2}\right\}$$

$$\text{ج) } y = \frac{x^2 + 1}{\sqrt{x^2 + 3}} \rightarrow \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \rightarrow \min \sqrt{x^2 + 3} = \sqrt{3}$$

$\sqrt{x^2 + 3} \neq 1$
 $\sqrt{x^2 + 3} \geq 0$

$$R_y = \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty\right)$$

$$\text{د) } y = x^2 + \frac{1}{x^2 + 4} \rightarrow x^2 + 4 + \frac{1}{x^2 + 4} - 4$$

$x^2 + 4 \neq 1 \rightarrow \min x^2 + 4 = 4$
 $x^2 + 4 \geq 0 \rightarrow \downarrow$

$$R_y = \left[\frac{17}{4}, +\infty\right)$$

$y = \frac{3x^2 + 4}{x^2 - 1}$ روش اصلی $\rightarrow yx^2 - y = 3x^2 + 4 \rightarrow 0 = (3-y)x^2 + y + 4 \rightarrow x = 0 \pm \frac{\sqrt{0 - 4(3-y)(y+4)}}{y-3}$

$y \neq 3$

$\Delta \geq 0 \rightarrow 4y^2 + 12y - 12y - 16 \geq 0$
 $4y^2 + 4y - 16 \geq 0$

$+ \left| - \right| + \left\{ R_y \leq (-\infty, -4] \cup (2, +\infty) \right\}$

$y = \frac{3x^2 + 4}{x^2 - 1}$ روش دوم $\rightarrow x^2 = 0 \rightarrow y = -4$

$x^2 = +\infty \rightarrow y = 3 \rightarrow \frac{3\infty + 4}{\infty - 1} \rightarrow y \neq 3$
 $R_y = (-\infty, 4] \cup (2, +\infty)$

$x^2 - 1$ می تواند صفر شود

$y = \frac{2\sin x - 1}{\sin x + 3}$ روش اصلی

$y \sin x + 3y = 2\sin x - 1 \rightarrow 3y + 1 = (2-y)\sin x$

$\rightarrow \sin x = \frac{3y+1}{2-y}$

$y \neq 2 \rightarrow -1 \leq \frac{3y+1}{2-y} \leq 1$

$\begin{cases} \frac{3y+1}{2-y} \leq 1 \rightarrow y \leq \frac{1}{5} \\ \frac{3y+1}{2-y} \geq -1 \rightarrow y \geq \frac{1}{5} \end{cases}$

$\begin{cases} y-2 \leq 3y+1 & y < 2 \rightarrow -\frac{3}{2} \leq y < 2 \\ y-2 \geq 3y+1 & y > 2 \rightarrow \emptyset \end{cases}$
 $-\frac{3}{2} \geq y$

$R_y = [-\frac{3}{2}, \frac{1}{5}]$

$y = \frac{2\sin x - 1}{\sin x + 3}$ روش دوم

$\sin x = -1 \rightarrow y = -\frac{4}{5}$

$\sin x = 1 \rightarrow y = \frac{1}{5}$

$R_y = [-\frac{4}{5}, \frac{1}{5}]$

$\sin x + 3$ نمی تواند صفر شود

$f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$

$\Delta < 0 \rightarrow$ خارج کننده ندارد $\rightarrow D_f = \mathbb{R}$

$f(x) = \frac{1(x^2 - x) + \frac{1}{4}}{1(x^2 - x) + 1}$

$x^2 - x = -\frac{1}{4} \rightarrow y = 0$

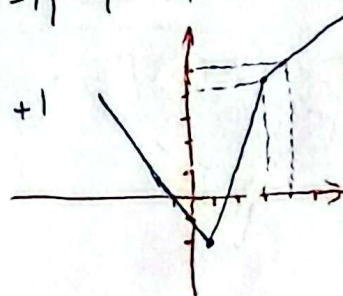
$x^2 - x = \infty \rightarrow y = 1 \rightarrow \frac{\infty + \frac{1}{4}}{\infty + 1} = 1$

$R_f = [0, 1)$

خارج کننده ندارد

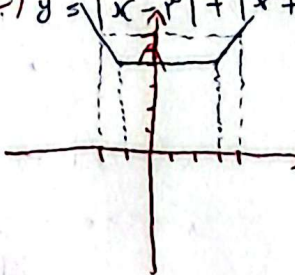
$y = |2x-2| - |x-3|$ (الف)

$-x-1 \left| \begin{matrix} 2x-0 \\ x+1 \end{matrix} \right| x+1$



$R_y = [-2, +\infty)$

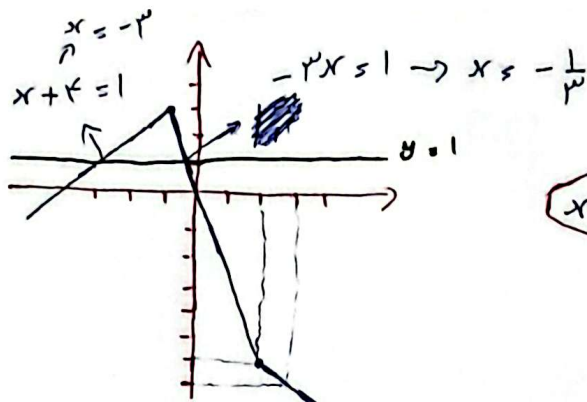
$y = |x-3| + |x+1|$ (ب)



$R_y = [4, +\infty)$

$$|x-2| - |2x+2| \leq 1$$

$$x+2 \left| -2x \right| -x-2$$



$$x \in (-\infty, -2] \cup [-\frac{1}{2}, +\infty)$$

الف) $y = [x] + [2-x]$
 $D_y = [1, 2]$
 $2 + [-x]$
 $2 + [x] + [-x]$

$$R_y = \{2, 3\}$$

ب) $y = 2x - [x]$
 $D_y = [-2, 2]$

$$R_y = [-2, 2)$$

$[2, 3) \rightarrow 2x+2$
 $[-1, 0) \rightarrow 2x+1$
 $[0, 1) \rightarrow 2x$
 $[1, 2) \rightarrow 2x-1$
 $\{2\} \rightarrow 2x-2$

الف) $y = \sin^2 x - \sin x + 1$

- $\sin x \leq 1 \rightarrow y \leq 1$
- $\sin x = -1 \rightarrow y = 3$
- $\sin x = \frac{b}{a} = \frac{1}{2} \rightarrow y = \frac{5}{4}$

$$R_y = [\frac{5}{4}, 3]$$

ب) $y = \sin^2 x + \sin^2 x + 1$

- $\sin^2 x \leq 1 \rightarrow y \leq 3$
- $\sin^2 x = 0 \rightarrow y = 1$
- $\sin^2 x = \frac{b}{a} = \frac{1}{2} \rightarrow$ ~~GOE~~

$$R_y = [1, 3]$$