به نام خدا

ماهان نژادکرد — تکلیف ۳ — دوازدهم پسر B

۱- $y = \dfrac{x^2+x+2}{x-1} \Rightarrow yx - y = x^2+x+2 \Rightarrow x^2 + x(1-y) + 2+y = 0$

$\xrightarrow{\text{روش } \Delta} x = \dfrac{y-1 \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \Rightarrow \Delta \geq 0 \Rightarrow 1+y^2-2y-4y-8 \geq 0$

$y^2-6y-7 \geq 0$

	-1		7	
$+$	0	$-$	0	$+$

$\Rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$

۲- الف) $y = 2x^2 - 5x + 1 \xrightarrow{a>0} y_v = \dfrac{-\Delta}{4a} = \dfrac{-17}{8}$

$\Rightarrow R_f = [-\frac{17}{8}, +\infty)$

ب) $y = \sqrt{\underbrace{-x^2+6x-5}}$

$a<0,\ y_v = \dfrac{-\Delta}{4a} = \dfrac{-16}{-4} = +4 \Rightarrow 0 \leq \sqrt{-x^2+6x-5} \leq 2$

$\Rightarrow -\infty < -x^2+6x-5 \leq +4$ $\qquad R_f = [0, 2]$

ج) $y = \sqrt{x^7 - 3x^2 + 4x + 1}$ $\qquad R_f = [0, +\infty)$

چون بزرگترین توان x فرد است پس برد عبارت زیر رادیکال برابر $\mathbb{R}$ است و که وقتی زیر رادیکال می‌رود تغییر می‌کند

$-\infty < x^7 - 3x^2 + 4x + 1 < +\infty \Rightarrow 0 \leq \sqrt{x^7 - 3x^2 + 4x + 1} < +\infty$

د) $\log(x^3 - 6x^2 + 7x + 4)$ $\qquad -\infty < x^3-6x^2+7x+4 < +\infty$ با توجه به توضیحات بالا:

$-\infty < \log(x^3-6x^2+7x+4) < +\infty \Rightarrow R_f = \mathbb{R}$

۳-

الف) $y = \dfrac{3x+1}{2x-4} \Rightarrow R_f = \mathbb{R} - \{\frac{a}{c}\} = \mathbb{R} - \{\frac{3}{2}\}$

Sol: $x = \dfrac{4y+1}{2y-3} \Rightarrow 2y-3 \neq 0 \Rightarrow y \neq \frac{3}{2} \Rightarrow R_f = \mathbb{R} - \{\frac{3}{2}\}$

ب) $y = \sqrt{\underbrace{\dfrac{4x+1}{x-2}}_{g(x)}}$ $\quad g(x) = \dfrac{4x+1}{x-2} \Rightarrow R_g = \mathbb{R} - \{\frac{a}{c}\} = \mathbb{R} - \{4\}$

$\Rightarrow f(x) = \sqrt{g(x)}$, $-\infty < g(x) < +\infty$, $g(x) \neq 4$ $\Big|$ $R_f = [0, +\infty) - \{2\}$

ج) $y = \dfrac{x^2+4}{\sqrt{x^2+3}} = \dfrac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \dfrac{1}{\sqrt{x^2+3}}$

$\sqrt{x^2+3} \geq 3 \Rightarrow y_{min} = \sqrt{3} + \dfrac{1}{\sqrt{3}} = \dfrac{4}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

$\Rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

د) $x^2+4 + \dfrac{1}{x^2+4} - 4$

$x^2+4 \geq 4 \Rightarrow y_{min} = 4 + \frac{1}{4} - 4 = \frac{1}{4} \Rightarrow R_f = [\frac{1}{4}, +\infty)$

if $a > 0$: $a + \frac{1}{a} \geq 2$ $\qquad$ سوالات ا/ب با این نکته حل شوند:

if $a < 0$: $a + \frac{1}{a} \leq -2$

$y = \dfrac{3x^2+4}{x^2-1} \Rightarrow x^2 = \dfrac{y+4}{y-3} \Rightarrow \dfrac{y+4}{y-3} \geq 0$ $\quad$ $-4 \quad 3$: $+ \; 0 \; - \; \| \; +$ $\quad -4$

$R_f = (-\infty, -4] \cup (3, +\infty)$

Sol: $x^2 \geq 0 \Rightarrow \begin{cases} x^2 = 0 \Rightarrow y = \dfrac{0+4}{0-1} = -4 \\ x^2 = +\infty \Rightarrow y = \lim\limits_{x^2 \to +\infty} \dfrac{3x^2+4}{x^2-1} = 3 \end{cases}$ $R_f = (-\infty, -4] \cup (3, +\infty)$

چون $x^2=1$ مخرج را صفر می‌کند

۵-

$$y=\frac{2\sin x-1}{\sin x+3}\ \rightarrow\ \sin x=-\frac{(3y+1)}{y-2}\ \rightarrow\ 1\geq\frac{3y+1}{y-2}\geq-1$$

$$0\leq\frac{3y+1}{y-2}+1=\frac{4y-1}{y-2}$$

$$\begin{array}{c|c|c} & \frac{1}{4} & 2 \\ \hline + & - & + \end{array}\ \rightarrow\ \left(-\infty,\frac{1}{4}\right]\cup\left(2,+\infty\right)\quad \mathrm{I}$$

$$\frac{3y+1}{y-2}-1\leq0\ \rightarrow\ \frac{2y+3}{y-2}\leq0$$

$$\begin{array}{c|c|c} & -\frac{3}{2} & 2 \\ \hline + & - & + \end{array}\ \rightarrow\ \left[-\frac{3}{2},2\right)\quad \mathrm{II}$$

$$\mathrm{I},\mathrm{II}:\ R_f=\left[-\frac{3}{2},\frac{1}{4}\right]$$

۲): $-1\leq\sin x\leq1\ \Rightarrow\ \begin{cases}\sin x=1\ \rightarrow\ y=\dfrac{2-1}{1+3}=\dfrac{1}{4}\\ \sin x=-1\ \rightarrow\ y=\dfrac{-2-1}{-1+3}=\dfrac{-3}{2}\end{cases}$

$$\sin x+3\neq0\ \rightarrow\ R_f=\left[-\frac{3}{2},\frac{1}{4}\right]$$

۶-

$$f(x)=\frac{x^2-x+\frac{1}{4}}{x^2-x+1}\ \rightarrow\ x^2-x+1\neq0\qquad \Delta=1-4=-3<0$$

$$\boxed{D_f=\mathbb{R}}$$

$$f(x)=\frac{x^2-x+1-\frac{3}{4}}{x^2-x+1}=1-\underbrace{\frac{3}{4(x^2-x+1)}}$$

$$\frac{3}{4}\leq x^2-x+1\ \Rightarrow\ 0<\frac{1}{x^2-x+1}\leq\frac{4}{3}\ \rightarrow\ 0<\frac{3}{4(x^2-x+1)}\leq1$$

$$1>1-\frac{3}{4(x^2-x+1)}\geq0$$

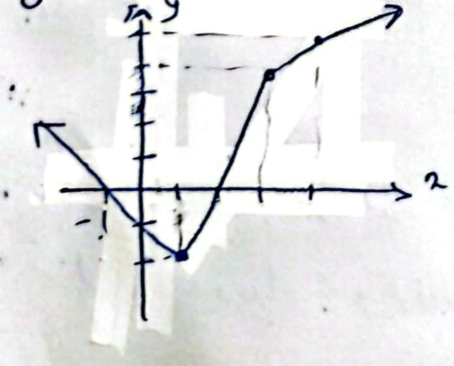
$$R_f=[0,1)$$

۷-

الف)

$$y=2|x-1|-|x-3|$$

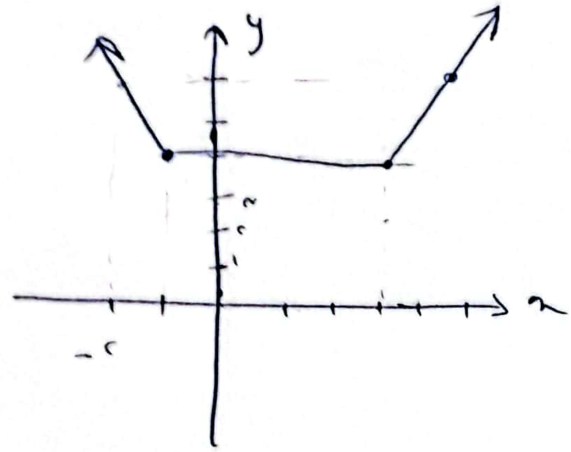
$$\begin{array}{c|c|c} & 1 & 3 \\ \hline -x-1 & 3x-5 & x+1 \end{array}$$



$$R_f=[-2,+\infty)$$

ب) $|x-3|+|x+1|$

-1		3
$-2x+2$	4	$2x-2$

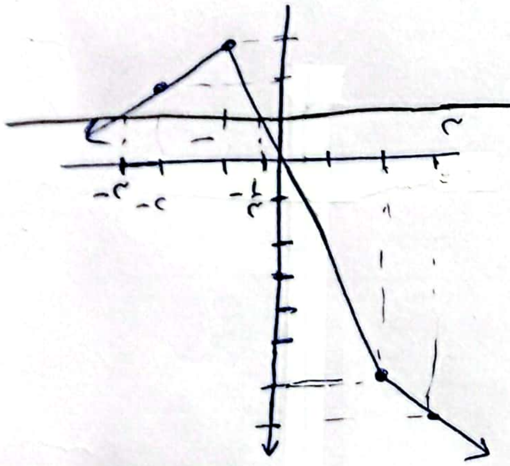


$R_f=[4,+\infty)$

۸-

$|x-2|-2|x+1|\leq 1$

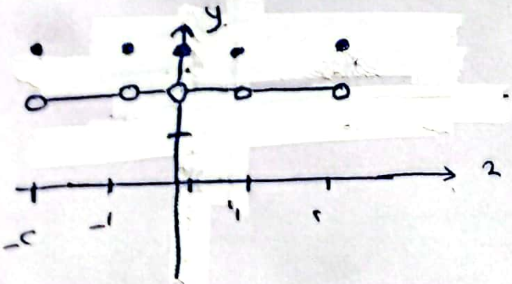
-1		2
$4+x$	$-3x$	$-x-4$



$y=1 \Rightarrow \begin{matrix} x\leq -3 \\ x\geq -\frac{1}{3} \end{matrix} \Rightarrow (-\infty,-3]\cup[-\frac{1}{3},+\infty)$

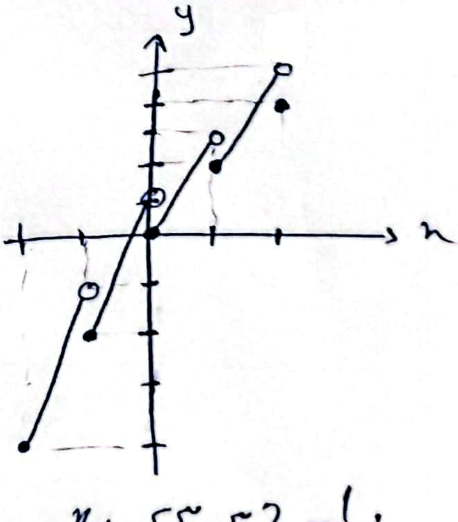
۹-

الف) $[x]+[2-x]=[x]+[-x]+2=\begin{cases} 2 & x\in\mathbb{Z} \\ 1 & x\notin\mathbb{Z} \end{cases}$



$R_f=\{1,2\}$

ب) $y=3x-[x]=\begin{cases} 3x+2 & -2\leq x<-1 \\ 3x+1 & -1\leq x<0 \\ 3x & 0\leq x<1 \\ 3x-1 & 1\leq x<2 \\ 4 & x=2 \end{cases}$



۱۰-

الف) $y=\sin^2 x-\sin x+1 \Rightarrow \begin{cases} \sin x=-1 \Rightarrow y=3 \\ \sin x=+1 \Rightarrow y=1 \\ \sin x=\frac{1}{2} \Rightarrow y=\frac{3}{4} \end{cases} \Rightarrow R_f=[\frac{3}{4},3]$

ب) $y=\sin^4 x+\sin^2 x+1 \Rightarrow \begin{cases} \sin^2 x=1 \Rightarrow y=3 \\ \sin^2 x=0 \Rightarrow y=1 \\ \sin^2 x=-\frac{1}{2} \times \end{cases} \Rightarrow R_f=[1,3]$