

$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + 2 + y = 0$$

$$\Delta \geq 0 \Rightarrow y^2 - 4y + 1 + 1 - 4y \geq 0 \Rightarrow y^2 - 4y - 2 \geq 0 \Rightarrow (y + 1)(y - 5) \geq 0$$

$$\Rightarrow y \geq 5, y \leq -1 \Rightarrow R_y = (-\infty, -1] \cup [5, +\infty)$$

$$y = 2x^2 - 5x + 1 \Rightarrow x = \frac{5 \pm \sqrt{25 - 8(1 - y)}}{4} \Rightarrow y \geq \frac{17}{8} \Rightarrow R_y = [\frac{17}{8}, +\infty)$$

$$y = \sqrt{-x^2 + 4x - 5} \Rightarrow f(x) = -x^2 + 4x - 5 \Rightarrow R_f = (-\infty, 4] \Rightarrow R_y = [0, 2]$$

$$y = \sqrt{x^2 - 2x + 5} \Rightarrow R_f = (-\infty, +\infty) \Rightarrow \sqrt{f(x)} \in [0, +\infty)$$

$$y = \log(x^2 - 4x + 7) \Rightarrow R_f = (-\infty, +\infty) \Rightarrow R_y = \mathbb{R}$$

$$y = \frac{x + 1}{x - 4} \Rightarrow x = \frac{y - 1}{-y + 4} \Rightarrow R_y = \mathbb{R} - \{\frac{4}{3}\}$$

$$y = \sqrt{\frac{x + 1}{x - 2}} \Rightarrow R_f = (-\infty, 2) \cup (2, +\infty) \Rightarrow R_y = [0, 2) \cup (2, +\infty)$$

$$y = \frac{x^2 + 1}{x^2 + 2} \Rightarrow y = \sqrt{x^2 + 2} + \frac{1}{\sqrt{x^2 + 2}} \Rightarrow R_y = [\sqrt{2} + \frac{1}{\sqrt{2}}, +\infty)$$

$$y = \frac{x^2 + 1}{x^2 + 2} - x \Rightarrow R_y = [0, +\infty) \Rightarrow \min y = \frac{1}{2} \Rightarrow R_y = [\frac{1}{2}, +\infty)$$

ریشه فردی: $y = \frac{cx^2 + 1}{x^2 - 1}$

ریشه زوجی: $y = \frac{cx^2 + 1}{x^2 - 1}$

$(c - y)x^2 + 1 + y = 0$

$\Delta \geq 0 \Rightarrow -(c - y)(1 + y) \geq 0$

$\Rightarrow y > c, y < -1$

$R_y = (-\infty, -1] \cup (c, +\infty)$

$R_y = (-\infty, -1] \cup (c, +\infty)$

اصول: $y \sin x + cy = 2 \sin x - 1$

$(y - 2) \sin x = -1 - cy$

$\sin x = \frac{-cy - 1}{y - 2}$

$-1 \leq \frac{-cy - 1}{y - 2} \leq 1 \Rightarrow -\frac{1}{y} \leq y \leq \frac{1}{y}$

منفی: $y = \frac{2 \sin x - 1}{\sin x + c}$

$\sin x \rightarrow 1 \Rightarrow \frac{1}{2}$

$\sin x \rightarrow -1 \Rightarrow -\frac{1}{2}$

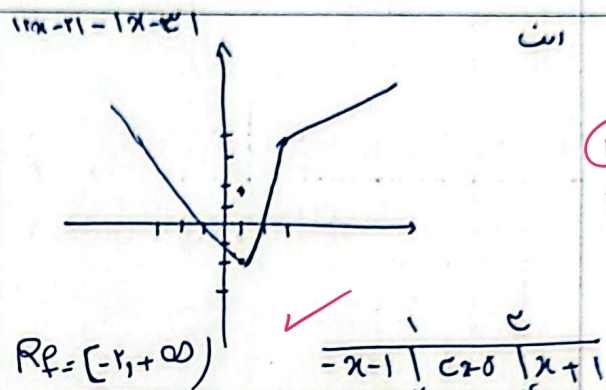
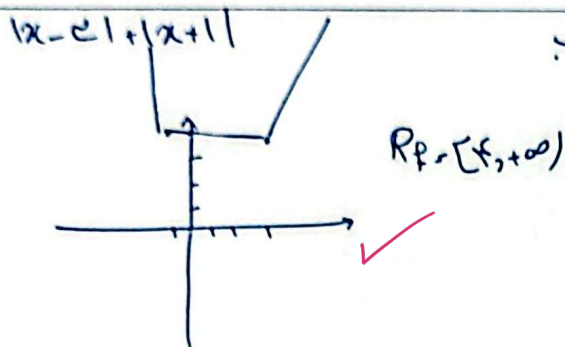
$R_y = [-\frac{1}{2}, \frac{1}{2}]$

$$D_f: x^2 - x + 1 \neq 0 \Rightarrow D_f: (-\infty, +\infty)$$

$$R_f: f(x) = \frac{x^2 - x + \frac{1}{2}}{x^2 - x + 1} = \frac{\frac{x^2 - x + 1}{2} + \frac{1}{2}}{x^2 - x + 1} = \frac{1}{2} + \frac{\frac{1}{2}}{x^2 - x + 1}$$

$x^2 - x \rightarrow +\infty \Rightarrow \frac{1}{2} = f(x)$
 $x^2 - x \rightarrow -\frac{1}{2} \Rightarrow \dots = f(x)$

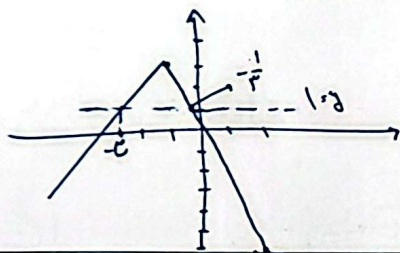
$$\text{مخرج صفری نه} \Rightarrow R_f = \left[\frac{1}{2}, \frac{5}{4} \right] \checkmark$$



$$|x-2| - |x+2| \leq 1$$

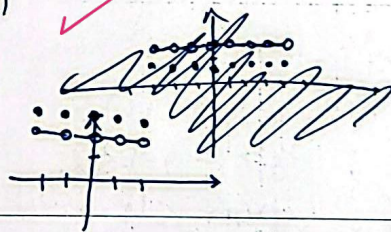
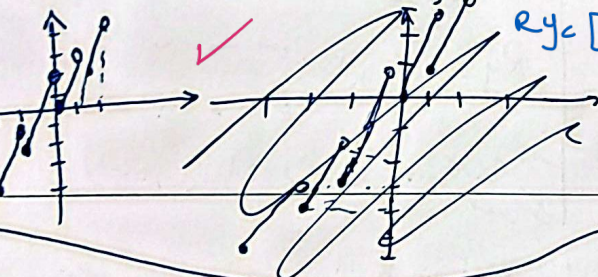
$$\frac{-1}{x+2} - \frac{2}{x+2} \leq -cx \quad \frac{2}{-4} - \frac{1}{-4} - x - 2$$

$$R_{f1} = (-\infty, -c) \cup \left[-\frac{1}{2}, +\infty\right) \checkmark$$



$[x] + [3-x]$
 $[x] + [-x] + 3$
 $x \in \mathbb{Z}: 3$
 $x \notin \mathbb{Z}: 2$

$3x - [x]$
 $-2 \leq x < -1 \rightarrow [x] = -2$
 $-1 \leq x < 0 \rightarrow [x] = -1$
 $0 \leq x < 1 \rightarrow [x] = 0$
 $1 \leq x < 2 \rightarrow [x] = 1$
 $2 \leq x < 3 \rightarrow [x] = 2$



الف) $\begin{cases} \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{2} \quad \frac{1}{2} + 1 = \frac{3}{2} \\ -1 \rightarrow 1 + 1 + 1 = 3 \\ 1 \rightarrow 1 \end{cases}$
 $R_{f2} = \left[\frac{3}{2}, 3 \right] \checkmark$

$\frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{2}$
 $\dots \rightarrow 1$
 $1 \rightarrow c$
 $[1, 3]$