نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲ کلاس دوازدهم ریاضی B

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$y = \frac{x^2+x+2}{x-1} \Rightarrow yx - y = x^2+x+2 \Rightarrow x^2 + (1-y)x + 2 + y = 0$

$\Delta \geq 0 \Rightarrow y^2 - 2y + 1 - 8 - 4y \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow (y+1)(y-7) \geq 0$

$\Rightarrow y \geq 7 \text{ , } y \leq -1 \Rightarrow R_y = (-\infty, -1] \cup [7, +\infty)$

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الف) $y = 2x^2 - 5x + 1 \Rightarrow x_s = \frac{5}{4} \Rightarrow y_s = 2 \times \frac{25}{16} - \frac{25}{4} + 1 = -\frac{17}{8} \xrightarrow{a>0} R_y = [-\frac{17}{8}, +\infty)$

ب) $y = \sqrt{-x^2+4x-5} \quad f = -x^2+4x-5 \rightarrow R_f = (-\infty, 4] \Rightarrow R_{\sqrt{f}} = [0, 2]$

ج) $y = \sqrt{\underbrace{x^4 - 6x^2 + 5x + 1}_{f(x)}} \longrightarrow R_f = (-\infty, +\infty) \Rightarrow \sqrt{f(x)} \in [0, +\infty)$

د) $y = \log(\underbrace{x^6 - 4x^2 + 7x + 5}_{f(x)}) \quad R_f = (-\infty, +\infty) \Rightarrow R_y = \mathbb{R}$

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الف) $y = \frac{2x+1}{2x-4} \longrightarrow x = \frac{-4y-1}{-2y+6} \Rightarrow R_y = \mathbb{R} - \{\frac{2}{2}\}$

ب) $y = \sqrt{\underbrace{\frac{5x+1}{x-2}}_{f(x)}} \Rightarrow R_f = (-\infty, 2) \cup (5, +\infty) \Rightarrow R_{\sqrt{f}} = [0, 2) \cup (5, +\infty)$

ج) $y = \frac{x^2+6}{\sqrt{x^2+6}} + \frac{1}{\sqrt{x^2+6}} \Rightarrow y = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+6}} \Rightarrow R_y = [\sqrt{6} + \frac{1}{\sqrt{6}}, +\infty)$

د) $y = \underbrace{x^2 + 5 + \frac{1}{x^2+4}}_{[4, +\infty)} - 4 \Rightarrow R_y = [0, +\infty)$

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روش اصلی:

$yx^2 - y = 6x + 4$

$(6-y)x^2 + 5 + y = 0$

$\Delta \geq 0 \Rightarrow -5(6-y)(5+y) \geq 0$

$\Rightarrow y \geq 6 \text{ , } y \leq -5$

$R_y = (-\infty, -4] \cup (3, +\infty)$

روش فرعی:

$y = \frac{6x^2+5}{x^2-1}$

$R_y = x^2 \nearrow +\infty \rightarrow 6$

$\searrow 0 \rightarrow -4$

مخرج جزی تواند صفر شود $\rightarrow (-\infty, -4] \cup [6, +\infty)$

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اصلی:

$y \sin x + 6y = 2 \sin x - 1$

$(y-2)\sin x = -1 - 6y$

$\sin x = \frac{-6y-1}{y-2}$

$-1 \leq \frac{-6y-1}{y-2} \leq 1 \Rightarrow -\frac{3}{2} \leq y \leq \frac{1}{4}$

فرعی:

$y = \frac{2\sin x - 1}{\sin x + 6}$

$\begin{cases} \sin x \to 1 & \frac{1}{7} \\ \sin x \to 0 & -\frac{3}{2} \end{cases}$

مخرج صفر نمی‌شود $\rightarrow R_y = [-\frac{3}{2}, \frac{1}{7}]$

$D_f : x^2-x+1 \neq 0 \Rightarrow D_f = (-\infty, +\infty)$ ۶
صفر نمی‌شود

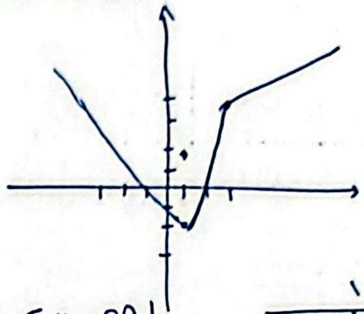
$R_f : f(x) = \dfrac{(x^2-x)+\frac{1}{4}}{(x^2-x)+1} = \dfrac{(\;)+\frac{1}{4}}{(\;)+1}$

$\begin{cases} x^2-x \to +\infty \Rightarrow 1 = f(x) \\ x^2-x \to -\frac{1}{4} \Rightarrow 0 = f(x) \end{cases}$

مخرج صفر نمی‌شه $\Rightarrow R_f = [0, 1)$

الف) ۷

$|2x-2| - |x-3|$

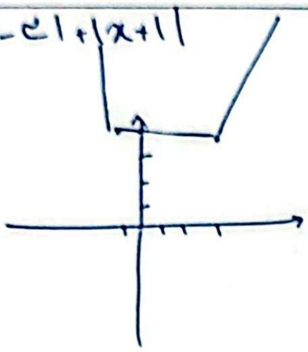


$R_f = [-2, +\infty)$

	1		3	
$-x-1$		$3x-5$		$x+1$
	-2		4	

ب)

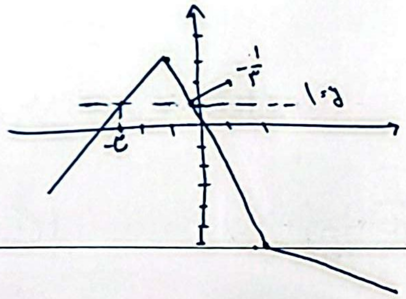
$|x-3| + |x+1|$



$R_f = [4, +\infty)$

$|x-2| - |2x+2| \leq 1$ ۸

	-1		2	
$x+4$		$-3x$		$-x-4$
	$+3$		-6	

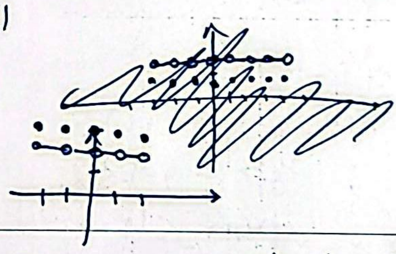


$(-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

الف) ۹

$[x] + [3-x]$

$[x] + [-x] + 3 \begin{cases} x \in \mathbb{Z} : 3 \\ x \notin \mathbb{Z} : 2 \end{cases}$



ب)

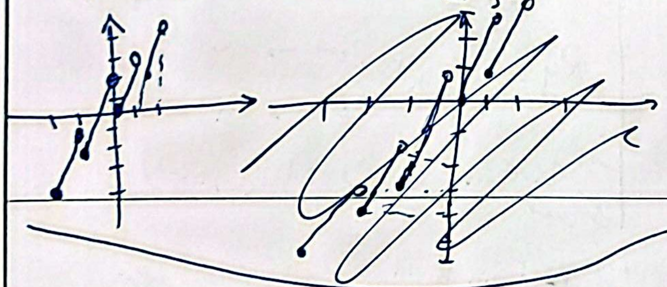
$3x - [x]$

$-2 \leq x < -1 \to [x] = -2$

$-1 \leq x < 0 \to [x] = -1$

$0 \leq x < 1 \to [x] = 0$

$1 \leq x < 2 \to [x] = 1$



الف) ۱۰

$\begin{cases} -\frac{b}{2a} = \frac{1}{2} \to \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4} \\ -1 \to 1+1+1 = 3 \\ 1 \to 1 \end{cases}$

$R_y = [\frac{3}{4}, 3]$

ب)

$\begin{cases} -\frac{b}{2a} = -\frac{1}{2} \to \text{[illegible]} \\ 0 \to 1 \\ 1 \to 3 \end{cases}$

$[1, 3]$