نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۳... کلاس ...[illegible]...

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$y = \frac{x^2+x+2}{x-1} \Rightarrow yx - y = x^2+x+2 \Rightarrow x^2+(1-y)x+y+2$

$\Rightarrow \Delta = y^2-2y+1-4y-8 \geq 0 \Rightarrow y^2-6y-7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0 \to y \geq 7, y \leq -1$

$\Rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$

۲

الف) $2x^2-5x+1 \Rightarrow -\frac{b}{2a} \Rightarrow \frac{5}{4} \Rightarrow 2 \times \frac{25}{16} - \frac{25}{4} + 1 \Rightarrow \frac{25}{8} - \frac{50}{8} + \frac{8}{8} = -\frac{17}{8} \Rightarrow y \geq -\frac{17}{8}$

ب) $y = \sqrt{-x^2+6x-5} \Rightarrow -\frac{b}{2a} = 3 \Rightarrow -9+18-5 = 4 \Rightarrow R_f = [0, 2]$

ج) $= \sqrt{x^4-3x^3+14x+1} \Rightarrow R_y^+ \cup \{0\}$

د) $\log(x^3-6x^2+7x+4) \Rightarrow x^3-6x^2+7x+4 > 0 \Rightarrow R_f = \mathbb{R}$

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$y = \frac{3x+1}{2x-6} \Rightarrow x = \frac{6y+1}{2y-3} \Rightarrow R_f \Rightarrow \mathbb{R} - \{\frac{3}{2}\}$

ب) $y = \sqrt{\frac{4x+1}{x-2}} \Rightarrow y^2 = \frac{4x+1}{x-2} \Rightarrow x = \frac{2y^2+1}{y^2-4} \Rightarrow R_f \Rightarrow [0, +\infty) - \{2\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} \Rightarrow y = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \Rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

د) $y = x^2 + \frac{1}{x^2+4}$

$\to y = x^2+4-4 + \frac{1}{x^2+4}$

$\to R_f = [\frac{1}{4}, +\infty)$

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$y = \frac{3x^2+4}{x^2-1} \Rightarrow x^2 = \frac{y+4}{y-3} \Rightarrow x = \sqrt{\frac{y+4}{y-3}} \Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

$yx^2 - y = 3x^2+4 \Rightarrow 3x^2 - yx^2 = -(y+4) \Rightarrow x^2(3-y) = -(y+4)$

$\Rightarrow x^2 = \frac{y+4}{y-3} \Rightarrow x = \sqrt{\frac{y+4}{y-3}} \Rightarrow R_f = (-\infty, -4]$

۵

$y = \frac{2\sin x - 1}{\sin x + 3} \Rightarrow \sin x = \frac{-3y+1}{y-2} \Rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$

$\to y\sin x + 3y = 2\sin x - 1 \Rightarrow (y-2)\sin x = -3y-1 \Rightarrow \sin x = \frac{3y+1}{y-2}$

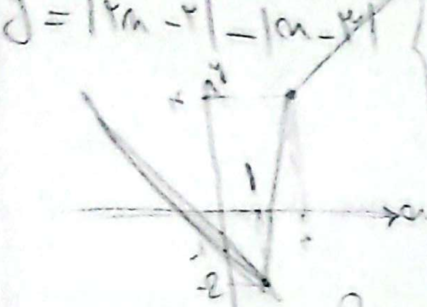
۶

$f(x) = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \Rightarrow x^2 - x + 1 \neq 0 \Rightarrow \Delta < 0 \Rightarrow D_f = \mathbb{R}$

$\Rightarrow f(x) = \dfrac{x^2 - x + 1 - \frac{3}{4}}{x^2 - x + 1} = 1 - \dfrac{3}{4x^2 - 4x + 4} \Rightarrow 0 < \dfrac{3}{4x^2 - 4x + 4} \leq 1 \Rightarrow R_f = [0, 1)$

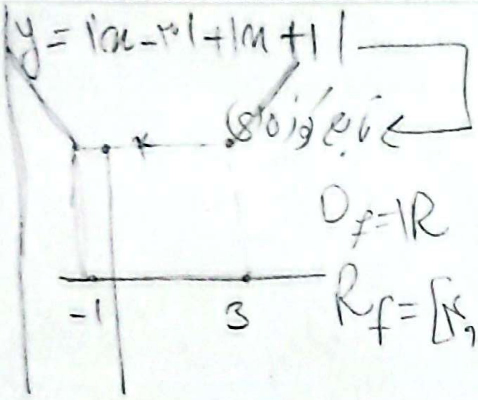
۷

$y = |2x - 2| - |x - 2|$ $\begin{cases} x \geq 2 \Rightarrow x \\ 1 \leq x < 2 \Rightarrow 3x \\ x < 1 \rightarrow -x \end{cases}$



$R_f = [-2, +\infty)$

$y = |x - 3| + |x + 1|$ تابع ثابت

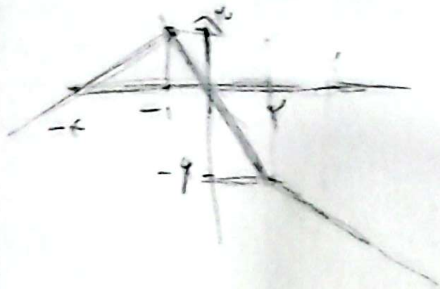


$D_f = \mathbb{R}$

$R_f = [4, +\infty)$

۸

$|x - 2| - |2x + 2| \leq 1$

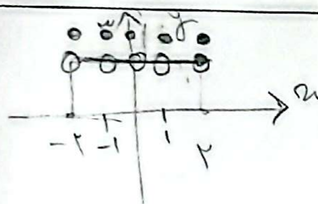


$\begin{cases} x \leq -1 \Rightarrow x + 4 \\ -1 \leq x \leq 2 \Rightarrow -x + 2 - 2x - 2 \Rightarrow -3x - 4 \\ x \geq 2 \Rightarrow -x - 4 \end{cases}$

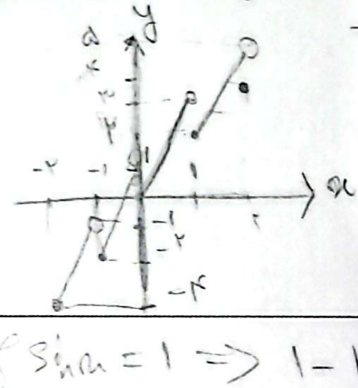
$R_f = (-\infty, 3] \cup [-\frac{1}{3}, +\infty)$

۹

$y = [x] + [3 - x] \Rightarrow y = [x] + 3 + [-x] \Rightarrow y = 3$



$y = 3x - [x]$



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$y = \sin^2 x - \sin x + 1 \Rightarrow \begin{cases} \sin x = 1 \Rightarrow 1 - 1 + 1 = 1 \\ -\frac{b}{2a} \Rightarrow \frac{1}{2} \Rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4} \Rightarrow [\frac{3}{4}, 3] \\ \sin x = -1 \Rightarrow 1 + 1 + 1 = 3 \end{cases}$

$y = \sin^4 x + \sin^2 x + 1$ $\begin{cases} \sin^2 = 1 \Rightarrow 1 + 1 + 1 = 3 \\ -\frac{b}{2a} = -\frac{1}{2} \Rightarrow \frac{1}{4} + \frac{1}{2} + 1 = \frac{7}{4} \Rightarrow [1, 3] \\ \sin^2 = 0 \Rightarrow 0 + 0 + 1 = 1 \end{cases}$