

$$yx - y = x^2 + x + 2 \Rightarrow x^2 + x - yx + 2 + y = x^2 + (1-y)x + 2 + y$$

$$x = \frac{y-1 \pm \sqrt{(1-y)^2 - 4(1)(2+y)}}{2} \quad y^2 - 2y + 1 - 4 - 4y = 0 \Rightarrow y^2 - 6y - 3 = 0$$

$$(y-1)(y+1) \geq 0 \quad \frac{-1}{+1} - \frac{1}{-1} + R_y = [1, +\infty) \cup (-\infty, -1]$$

$$\text{الف) } x_3 = \frac{-b}{2a} = \frac{0}{4} = 0 \quad y_3 = 2\left(\frac{0}{4}\right)^2 - 2\left(\frac{0}{4}\right) + 1 = 2 \times \frac{0}{16} - \frac{0}{8} + 1 = \frac{0}{16} - \frac{0}{16} + \frac{16}{16} = \frac{16}{16} = 1$$

$$R = \left[-\frac{1}{4}, +\infty\right)$$

$$\text{ب) } x_3 = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad y_3 = -9 + 11 - 0 = 2 \quad (-\infty, 2] \Rightarrow [0, 2] = R \dots$$

$$\text{ج) } \sqrt{1-x^2} \geq 0 \Rightarrow R = [0, +\infty) = R$$

د)

$$\text{الف) } R = R - \left\{\frac{a}{c}\right\} \Rightarrow R_y = R - \left\{\frac{1}{2}\right\}$$

$$\text{ب) } R_y = R - \{1\} \Rightarrow R_y = [0, +\infty) - \{1\}$$

$$\text{ج) } \frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3} \neq 1 \Rightarrow \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty\right)$$

$$\text{د) } x^2+4+\frac{1}{x^2+4} - 4 \Rightarrow \left[4+\frac{1}{4} - 4, +\infty\right) \Rightarrow \left[\frac{1}{4}, +\infty\right)$$

$$yx^2 - y = 3x^2 + 4 \Rightarrow (y-3)x^2 = 4+y \Rightarrow x^2 = \frac{4+y}{y-3} \Rightarrow \frac{4+y}{y-3} \geq 0 \quad \frac{-4}{+4} \leq \frac{3}{-3}$$

$$R_x = (-\infty, +\infty) \cup (-\infty, -4]$$

$$x^2 = 0 \Rightarrow -4 = y$$

$$x^2 = +\infty \Rightarrow 3 = y \Rightarrow (3, +\infty) \cup (-\infty, -4]$$

$$\frac{2 \sin x - 1}{\sin x + 3} \geq 0 \Rightarrow \sin x = \frac{3y+1}{y-2} = \frac{-3y-1}{+y-2}$$

$$-1 \leq \frac{-3y-1}{y-2} \leq 1 \Rightarrow \begin{cases} \frac{-3y-1}{y-2} \leq 1 \\ \frac{-3y-1}{y-2} \geq -1 \end{cases} \Rightarrow \begin{cases} \frac{-3y-1}{y-2} \leq 1 \\ \frac{-3y-1}{y-2} \geq -1 \end{cases} \Rightarrow \begin{cases} \frac{-3y-1}{y-2} \leq 1 \\ \frac{-3y-1}{y-2} \geq -1 \end{cases}$$

$$\frac{-3y-1}{y-2} \geq -1 \Rightarrow \frac{-3y-1}{y-2} + \frac{y-2}{y-2} = \frac{-2y-3}{y-2} \geq 0$$

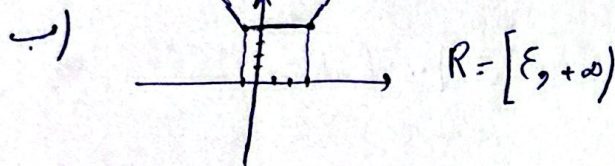
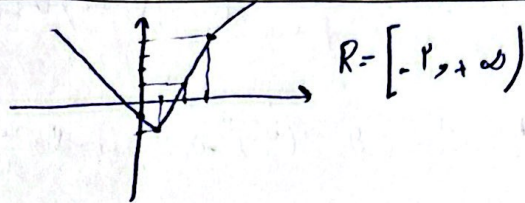
$$\frac{-3}{-1} + \frac{1}{1} = \frac{-3+1}{-1+1} = \frac{-2}{0} \Rightarrow R = \left[\frac{3}{2}, +\infty\right)$$

$$\frac{-3y-1}{y-2} \leq 1 \Rightarrow \frac{-3y-1}{y-2} - \frac{y-2}{y-2} = \frac{-4y+1}{y-2} \leq 0 \Rightarrow \frac{-4}{1} + \frac{1}{-1} = \frac{-4-1}{-1+1} = \frac{-5}{0} \Rightarrow R = (-\infty, \frac{1}{4}]$$

$$f(x) = \frac{(x - \frac{1}{x})^2}{(x - \frac{1}{x})^2 + \frac{1}{x^2}} \Rightarrow \begin{aligned} x - \frac{1}{x} &\rightarrow +\infty \Rightarrow 1 \\ x - \frac{1}{x} &\rightarrow 0 \Rightarrow 0 \end{aligned}$$

$$R_f = [0, 1]$$

الف) 
$$\begin{array}{c|c|c} 1 & & 3 \\ \hline -x^2 + x + x - 3 & x^2 - x + x - 3 & x^2 - x - x + 3 = x^2 - 2x + 3 \\ \hline & = x^2 - 2x & \end{array}$$
  
 $x = -x - 1$



$$\begin{array}{c|c|c} -1 & & 2 \\ \hline x - x + x^2 + x = x^2 + x & x^2 - x + x^2 + x = 2x^2 & x^2 - x - x^2 - x = -2x - 2 < 1 \Rightarrow -x < 0 \Rightarrow x > 0 \Rightarrow x > 2 \\ \hline & = -x^2 & \end{array}$$
  
 $x + 1 < 1 \Rightarrow x < -1$   
 $x < -1$   
 $x > 2$   
 $\Delta_{x^2 - x - 2} = 1 + 8 = 9$   
 $x = \frac{1 \pm 3}{2} = -1, 2$   
 $(-\infty, -1] \cup [\frac{1}{2}, 2] \cup [2, +\infty)$

الف) 
$$\begin{aligned} \sin x = 1 &\Rightarrow 1 - 1 + 1 = 1 \\ \sin x = -1 &\Rightarrow 1 - (-1) + 1 = 3 \\ \sin x = \frac{1}{x} &\Rightarrow \frac{1}{x} - \frac{1}{x} + 1 = \frac{1}{x} \end{aligned} \Rightarrow \left[\frac{1}{x}, 3\right] \subset R$$

ب) 
$$\begin{aligned} \sin^2 x = 0 &\Rightarrow 1 \\ \sin^2 x = 1 &\Rightarrow 3 \\ \sin^2 x = \frac{1}{x} &\Rightarrow \frac{1}{x} \end{aligned} \Rightarrow R = [1, 3]$$